# PRINCIPLES AND TECHNIQUES OF MODERN RADAR SYSTEMS

# Lecture Notes

**III B.Tech – II Semester – R19 Regulation**

**ELECTRONICS & COMMUNICATION ENGINEERING**


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**B.Tech (ECE) – III-II Sem**

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# PRINCIPLES AND TECHNIQUES OF MODERN RADAR SYSTEMS - (19A04605e)

**Professional Elective-II**

## Course Objectives:

- *To understand the basic principles of RADAR and its variants, RADAR based Microwave imaging.*
- *To apply the fundamental knowledge of various RADARs, Matched Filter and to find the range between the target and RADAR, frequency and phase of the received signal.*
- *To analyze the received data from the target using CW RADAR & MTI RADAR andto find the distance, tracking range for clutter analysis.*

**Unit 1:** Basic Principles: Fundamental elements of Radar and its block diagram, Radar equation – Signal to Noise Power Ratio (SNR), Radar Cross section – Cross sections of small targets, Examples of target cross sections, cross section fluctuations and models.

**Unit 2**: CW Radar – Principle, block diagram, FMCW Radar, Pulsed Radar Principles, Clutter Analysis, MTI Improvement Factor, Pulsed Doppler Radar, range measurement.

**Unit 3:** Tracking in Radar, Frequency measurement and tracking, Angular resolution, Monopulse Technique, Detection Theory: Match Filtering, Radar Ambiguity Function.

**Unit 4**: Imaging Radar: Resolution Concept, Pulse Compression, Synthetic Aperture Processing, ISAR Imaging, Probability of false alarm and Detection, Modified Radar Range Equation with Swerling Models.

**Unit 5**: Ground Penetrating Radar for close sensing**,** Radar Tomography and Radar based Microwave Imaging, Emerging and Modern Applications of Radar Principles.

## Unit - 01

# BASIC PRINCIPLES

**RADAR** is an abbreviation for **RA**dio **D**etection **A**nd **R**anging. A system used for detecting and locating the presence of objects like ships, vehicles, aircraft etc. by radiating electromagnetic signal in space is known as the **Radar system**.

Basically, radar is used to collect the information related to the object or target like its range and location by radiating electromagnetic energy and examining the echo received from the distant object.

## History:

Radar was invented for military purpose before **world war II** in order to secretly detect the presence of unknown objects. Initially, the transmitting tubes were not that much powerful thus worked at a very low frequency of about **60 MHz**.

But further development in the field and use of magnetrons has extended the frequency range to a higher level.

Radar system

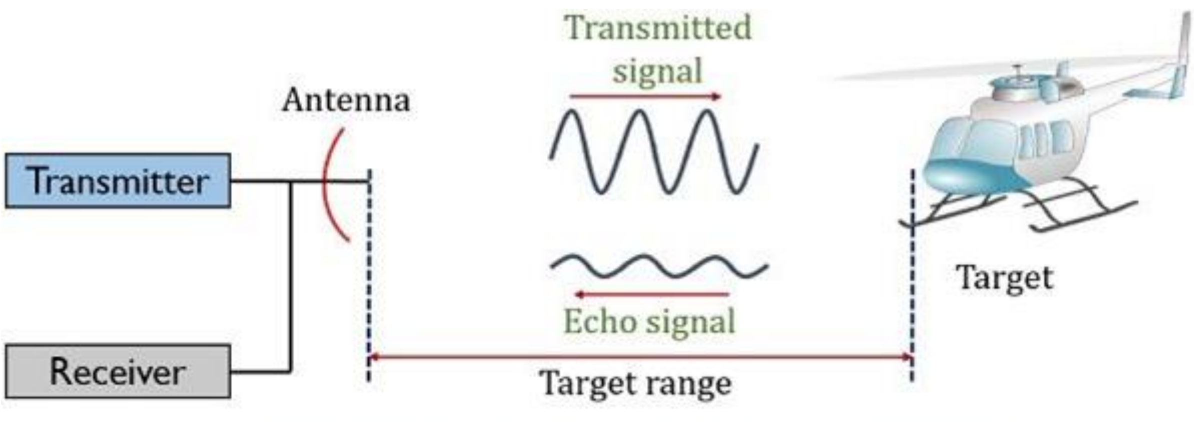
According to the operation performed by the radar, it is very important to have a system that can accurately detect the presence of the target. So for this purpose, narrow beam antennas with short-wavelength are used that correspond to upper UHF and microwave frequencies.

Thus the US army developed microwave radar system and such a system can determine the position of the object to within **0.1°** and **25 meters**.

## Principle:

A radar system operates in a way that it radiates electromagnetic energy into space and detects various aspects related to objects by analysing the echo generated when the radiated energy gets re-radiated by the object.

The figure below shows the basic principle of radar:



Basic Principle of Radar

The electromagnetic signal is produced by the transmitter unit and is radiated in space by the radar antenna. While the receiver performs extraction of information from the signal received by the radar antenna.

We know whenever an electromagnetic wave is transmitted by the system then it reflects or re-radiates some of its parts on experiencing a variation in the conductivity of the medium. This variation in conductivity arises due to the presence of an object either stationary or moving. Thereby producing an echo.

The radar system receives the echo by the help of an antenna in order to analyse it and have the location of the object.

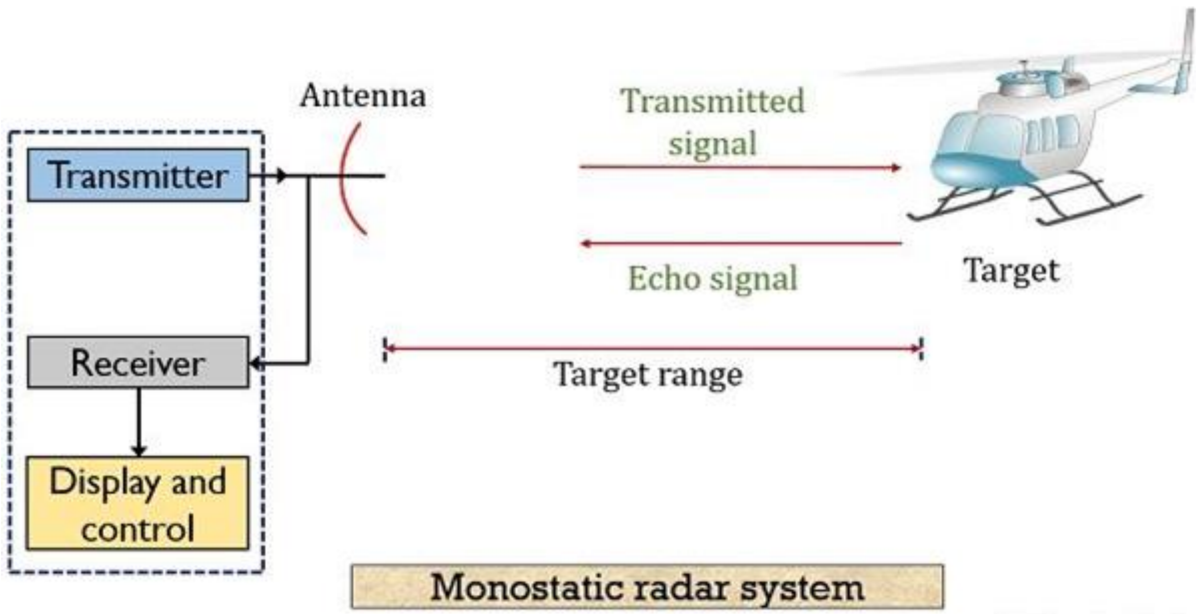
Range specifies the distance between the target and the radar system.

The range to an object is determined by the measurement of the time taken by the radiated signal to reach the object and come back to the radar. And the location of the stationary object in the space is determined from the angle pointed by the antenna when the echo received is of maximum amplitude.

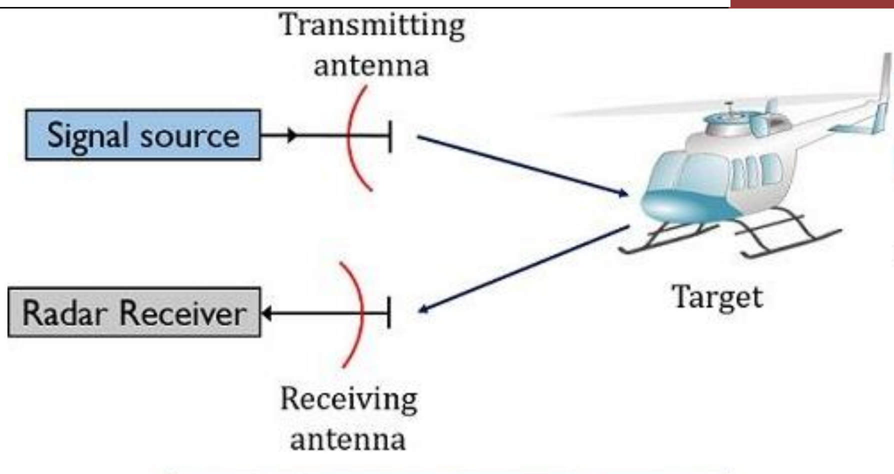
For a moving object because of the Doppler effect, there exists a shift in the frequency of the re-radiated signal. And the frequency shift shows proportionality with the radial velocity of the object.

Basically, there exist two major radar systems:

**Monostatic Radar System**: A monostatic radar system uses a single antenna for transmission as well as reception purpose.



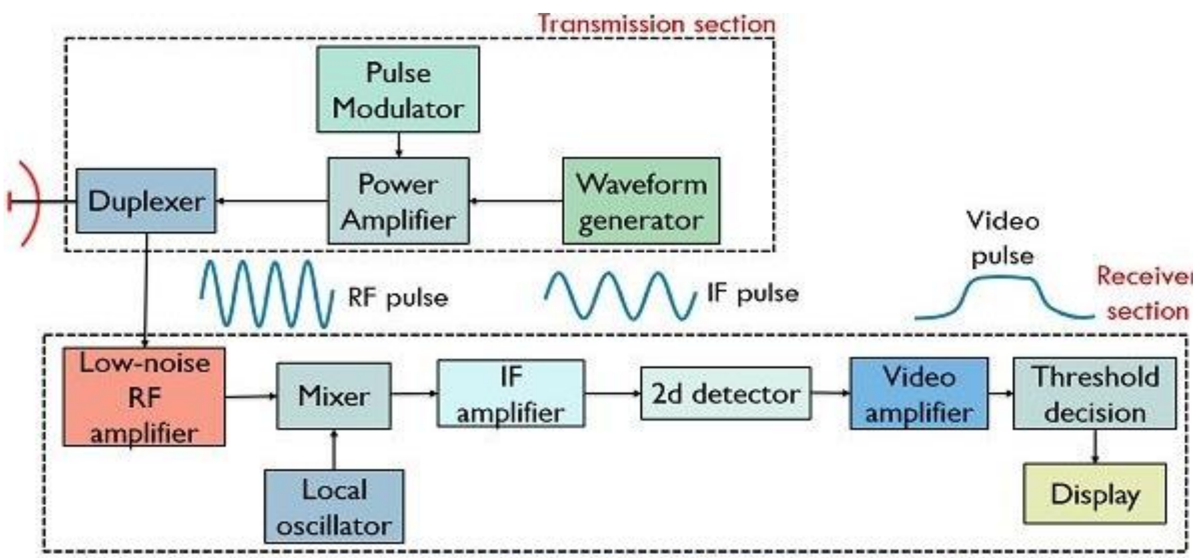

**Bistatic Radar System**: A bistatic radar system utilizes independent antennas for transmission and reception of the signal.



Bistatic radar system

## Block Diagram of Radar System:

The figure below shows the block diagram representation of radar:



Block Diagram of Radar system

We know that a radar system has a transmitting and receiving section. And both the sections perform their respective operation.

Let us now discuss how radar operates:

***Transmitter Section:*** The transmitter section is composed of the following units:

**1. Waveform Generator**: The waveform generator (usually a magnetron) generates a radar signal at low power which is to be transmitted into space.

**2. Transmitter**: The signal generated by the waveform generator is fed to the transmitter. The transmitter section can be a magnetron, travelling wave tube or a transistor amplifier. In the case of pulse radar, magnetrons are widely used as transmitters but whenever there exists a need for high average power then amplifiers are used.

**3. Pulse modulator**: A pulse modulator is used to build synchronization between the waveform generator and transmitter.

The pulse modulator causes the turning on and off of the power amplifier according to the input pulses generated by the waveform generator.

**4. Duplexer**: A duplexer is basically used to form isolation between transmitter and receiver section. A duplexer allows the use of a single antenna for both transmission and reception purpose. However, both the sections operate at different power level, therefore, a duplexer is used to isolate the two section.

Thus the signal from the transmitter is provided to the antenna through the duplexer. As the duplexer short circuits the input of the receiver section.

Also, the re-radiated signal received by the common antenna is fed to the receiver section using duplexer.

***Receiver Section:*** The following components are present inside the receiver section:

**5. Low noise RF amplifier**: The receiver must be superheterodyne. The unit acts as the input stage for the receiver section. The RF amplifier generates an RF pulse which is proportional to the echo of the transmitted signal.

**6. Mixer and Local Oscillator**: The RF pulse received from the low noise RF amplifier is converted into an IF pulse. Usually, the RF amplifier acts at the input stage of the receiver section but sometimes the mixer acts at the input stage by eliminating the RF amplifier. But this leads to a less sensitive receiving section due to the high noise figure of the mixer.

**7. IF amplifier**: The IF pulse generated by the mixer circuit is amplified by the IF amplifier. It acts as a matched filter and increases the SNR of the received signal. Also, it enhances the echo detecting ability of the receiver section by reducing the effects of unwanted signals. The receiver’s bandwidth is associated with the bandwidth of the IF stage.

**8. 2$^{nd}$ Detector or Demodulator**: This unit is nothing but a crystal diode that performs demodulation of the signal by separating the transmitted signal from the carrier.

**9. Video Amplifier**: This unit amplifies the received signal to a level that can be displayed on the screen.

**10. Threshold decision**: This unit makes the decision about the existence of the target in space. Basically, it has some threshold limit set which is compared with the magnitude of the received signal.

If the threshold value is surpassed by the output signal, then this shows that presence of the target. Otherwise, it is assumed that only the noise component is present in the space.

**11. Display**: The display unit shows the final output of the receiver section. **PPI** i.e., plan position indication is typically used as the radar display unit.

It presents the range and location of the object by mapping it in polar coordinates. PPI is implemented with CRT.

The output signal modulates the electron beam of the cathode ray tube in order to permit the electron beam to sweep from the centre in the outward direction of the tube. And this sweep shows rotation in synchronization with the pointing of the antenna.

## Applications of Radar:

Radar systems find its applications in a wide variety of fields like military, air traffic control, in weather forecasting, remote sensing, astronomy, mapping etc.

1. **Military**: It is the major application of radar and is one of the most important parts of the air defence system. Radar is used for the purpose of navigation and surveillance in the military for secure operations.
2. **Air traffic controlling**: Radar is used to control the air traffic in the air routes and airports. High-resolution radars are used for analysing the aircraft and ground vehicular traffic at the airports.
3. **Ship safety**: Radars are used to provide safety measures to the ships in bad visibility conditions by giving alerts about the existence of other ships in the route.
4. **Remote sensing**: Radar is a remote sensor by nature as they can sense the geophysical objects. And these are used forecasting of weather conditions along with agricultural conditions and environmental pollution.

## Maximum Unambiguous Range:

A problem with pulsed radars and range measurement is how to unambiguously determine the range to the target if the target returns a strong echo. This problem arises because of the fact that pulsed radars typically transmit a sequence of pulses. The radar receiver measures the time between the leading edges of the last transmitting pulse and the echo pulse. It is possible that an echo will be received from a long range target after the transmission of a second transmitting pulse.

In this case, the radar will determine the wrong time interval and therefore the wrong range. The measurement process assumes that the pulse is associated with the second transmitted pulse and declares a much reduced range for the target. This is called range ambiguity and occurs where there are strong targets at a range in excess of the pulse repetition time. The pulse repetition time defines a maximum unambiguous range. To increase the value of the unambiguous range, it is necessary to increase the PRT, this means: to reduce the PRF.

Echo signals arriving after the reception time are placed either into the

- transmit time where they remain unconsidered since the radar equipment isn't ready to receive during this time, or
- into the following reception time where they lead to measuring failures (ambiguous returns).

$$\mathbf{R_{un}} = PRT * c/2$$

**Eg:** A Pulse Radar is operating at a frequency of 1.2 G Hz and PRF of 330 Hz. Find out its Maximum unambiguous Range.

$PRT = 1/PRF = 1/330 = 0.00303$ s

$\mathbf{R_{un}} = PRT * c/2 = 0.00303 \times 3 \times 10^8 /2 = 0.00454545 \times 10^8$ m $= 454.54$ km

## The Simple Form of the Radar Equation:

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target, and environment. It is useful not just as a means for determining the maximum distance from the radar to the target, but it can serve both as a tool for understanding radar operation and as a basis for radar design.

If the power of the radar transmitter is denoted by **Pt** and if an isotropic antenna is used (one which radiates uniformly in all directions), the **power** density (watts per unit area) at a distance **R** from the radar is equal to the transmitter power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R, or

Power density from isotropic antenna = $P_t / 4\pi R^2$

Radars employ directive antennas to direct, the radiated power **Pt** into some particular direction. The **gain** G of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna.

The power density at the target from an antenna with a transmitting gain G is,

Power density from directive antenna = $P_t G / 4\pi R^2$

The target intercepts a portion of the incident power and reradiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section **σ** and is defined by the relation

Power density of the echo signal at the radar = $P_t G \sigma / (4\pi R^2)^2$

The radar antenna captures a portion of the echo power. If the effective area of the receiving antenna is denoted **Ae,** the power **Pr** received by the radar is

$$P_r = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} A_e = \frac{P_t G A_e \sigma}{(4\pi)^2 R^4}$$

The maximum radar range **Rmax** is the distance beyond which the target cannot be detected. It occurs when the received echo signal power **Pr** just equals the minimum detectable signal **Smin.**

Therefore

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}$$

Antenna theory gives the relationship between the transmitting gain and the receiving effective area of an antenna as

$$G = \frac{4\pi A_e}{\lambda^2}$$

Since radars generally use the same antenna for both transmission and reception, Above equation Can be substituted first for **Ae** then for G, to give two other forms of the radar equation

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4}$$

$$R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4}$$

## Radar Resolution:

The target resolution of a radar is its ability to distinguish between targets that are very close in either range or bearing. Weapons-control radar, which requires great precision, should be able to distinguish between targets that are only yards apart. Search radar is usually less precise and only distinguishes between targets that are hundreds of yards or even miles apart. Radar resolution is usually divided into two categories; range resolution and angular (bearing) resolution.

***Range Resolution***

Range resolution is the ability of a radar system to distinguish between two or more targets on the same bearing but at different ranges. The degree of range resolution depends on the width of the transmitted pulse, the types and sizes of targets, and the efficiency of the receiver and indicator.

Pulse width is the primary factor in range resolution.

$S_r = c\, P_W / 2$

Where; $c$ = speed of light

$P_W$ = transmitters pulse width

$S_r$ = range resolution as a distance between the two targets

***Angular Resolution***

Angular resolution is the minimum angular separation at which two equal targets at the same range can be detected separately. The half-power points of the antenna radiation pattern (i.e. the -3 dB beam width) are normally specified as the limits of the antenna beam width for the purpose of defining angular resolution; two identical targets at the same distance are, therefore, resolved in angle if they are separated by more than the antenna beam width. An important remark has to be made immediately: the smaller the beam width Θ, the higher the directivity of the radar antenna, the better the bearing resolution.

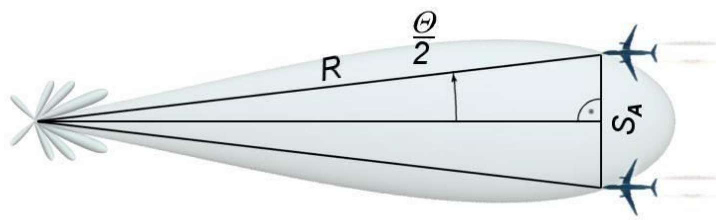



**Figure:** Angular resolution

The angular resolution as a distance between two targets depends on the slant-range and can be calculated with help of the following formula:

$$S_A \leqq 2R \cdot \sin\frac{\Theta}{2} \; [m]$$

Where

θ = antenna beam width

SA = angular resolution as a distance between the two targets

R = slant range in m

## Radar Bands and Usage:

| Band Designation | Frequency Range | Usage |
|---|---|---|
| HF | 3–30 MHz | OTH surveillance |
| VHF | 30–300 MHz | Very-long-range surveillance |
| UHF | 300–1,000 MHz | Very-long-range surveillance |
| L | 1–2 GHz | Long-range surveillance<br>En route traffic control |
| S | 2–4 GHz | Moderate-range surveillance<br>Terminal traffic control<br>Long-range weather |
| C | 4–8 GHz | Long-range tracking<br>Airborne weather detection |
| X | –12 GHz | Short-range tracking<br>Missile guidance<br>Mapping, marine radar<br>Airborne intercept |
| $K_u$ | 12–18 GHz | High-resolution mapping<br>Satellite altimetry |
| K | 18–27 GHz | Little use (water vapor) |
| $K_a$ | 27–40 GHz | Very-high-resolution mapping<br>Airport surveillance |
| millimeter | 40–100+ GHz | Experimental |

## Minimum Detectable Signal:

The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy. **The weakest signal the receiver can detect is called the *minimum detectable signal.***

Detection is based on establishing a threshold level at the output of the receiver. If the receiver output exceeds the threshold, a signal is assumed to be present. This is called ***threshold detection.***

This might represent one sweep of the video output displayed on an A-scope. The envelope has a fluctuating appearance caused by the random nature of noise. If a large signal is present such as at ***A*** in Fig. 2.1, it is greater than the surrounding noise peaks and can be recognized on the basis of its amplitude. Thus, if the threshold level were set sufficiently high, the envelope would not generally exceed. the threshold if noise alone were present, but would exceed it if a strong signal were present.

If the signal were small, however, it would be more difficult to recognize its presence. The threshold level must be low if weak signals are to be detected, but it cannot be so low that noise peaks cross the threshold and give a false indication of the presence of targets.

## Receiver Noise:

Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal. It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. Since noise is the chief factor limiting receiver sensitivity, it is necessary to obtain some means of describing it quantitatively.

If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is called thermal noise, or Johnson noise, and is directly proportional to the temperature of the ohmic portions of the circuit and the receiver bandwidth. The available thermal-noise power generated by a receiver' of bandwidth ***B,*** (in hertz) at a temperature **T** (degrees Kelvin) is equal to

Available thermal-noise power = **kTB,** ***(2.2)***

Where k is boltzmann's constant = $1.38 \times 10^{-23}$ J/deg

A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic loss. The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.

For radar receivers of the super heterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the **intermediate- frequency** stages. It should be cautioned that the bandwidth ***B,*** of Eq. (2.2) is not the 3-dB, or half-power, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2} \qquad (2.3)$$

where H( f ) = frequency-response characteristic of IF amplifier (filter) and

$f_o$ = frequency of maximum response (usually occurs at midband).

When ***H(f)*** is normalized to unity at midband (maximum-response frequency),

$H(f_o) = 1$.

The bandwidth $B_n$ is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is the same as the filter with characteristic ***H(f).*** The 3 db bandwidth is defines as separation in Hertz between the points on the frequency response characteristic where the response is reduced to 0.707 (3 dB) from its maximum value. The 3 db bandwidth is widely used as it is easy to measure. However, involves a complete knowledge of the response characteristic ***H(f).*** The frequency - response characteristic of many practical radar receivers are such that the **3-dB** and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.

The noise power in practical receivers is often greater than can be accounted for by thermal noise alone. The additional noise components are due to mechanisms other than the thermal agitation of the conduction electrons. For purposes of the present discussion, however, the exact origin of the extra noise components is not important except to know that it exists. No matter whether the noise is generated by a thermal mechanism or by some other mechanism. the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an " ideal " receiver multiplied by a factor called the ***noise figure.*** The noise figure $F_n$ of a receiver is defined by the equation

$$F_n = \frac{N_o}{kT_0 B_n G_a} = \frac{\text{noise out of practical receiver}}{\text{noise out of ideal receiver at std temp } T_0} \qquad (2.4a)$$

where $N_o$ = Noise output from receiver, and G, = available gain.

The standard temperature $T_o$ is taken to be 290 K**,** according to the Institute of Electrical and Electronics Engineers definition.

The noise $N_o$ is measured usually at the output of the IF amplifier before the nonlinear second detector. 'The receiver bandwidth $B_n$ is that of the IF amplifier in most receivers. The available gain G, is the ratio of the signal out $S_o$ to the signal in $S_i$**,** and $kT_oB_n$ is the input noise $N_i$ in an ideal receiver. Equation (2.4a) may be rewritten as

$$F_n = \frac{S_i/N_i}{S_o/N_o} \qquad (2.4b)$$

$$N_i = kT_oB_n$$

$$S_i = \frac{kT_0B_nF_nS_o}{N_o} \qquad (2.5)$$

If the minimum detectable signal $S_{min}$ is that value of $S_i$ corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o / N_o)_{min}$ necessary for detection, then

$$S_{min} = kT_0B_nF_n\left(\frac{S_o}{N_o}\right)_{min} \qquad (2.6)$$

Substituting **Eq.** (2.6) into Eq. (2.1) results in the following form of the radar equation:

$$R_{max}^4 = \frac{P_tGA_e\sigma}{(4\pi)^2kT_0B_nF_n(S_o/N_o)_{min}} \qquad (2.7)$$

In the above radar range equation the $S_{min}$ is replaced by $(S_o / N_o)_{min}$. The advantage is $(S_o / N_o)_{min}$ is independent of receiver bandwidth and Noise Figure. $(S_o / N_o)_{min}$ is that if the IF Amplifier.

## Determination of Signal to Noise Ratio $(S_o / N_o)_{min}$:

Both the false-alarm time and the detection probability are specified by the system requirements. The radar designer computes the probability of the false alarm and the Fig. below determines the signal-to-noise ratio. This is the signal-to-noise ratio that is used in the equation for minimum detectable signal [Eq. **(2.6)]**. The signal-to-noise ratios of below Fig. apply to a single radar pulse.

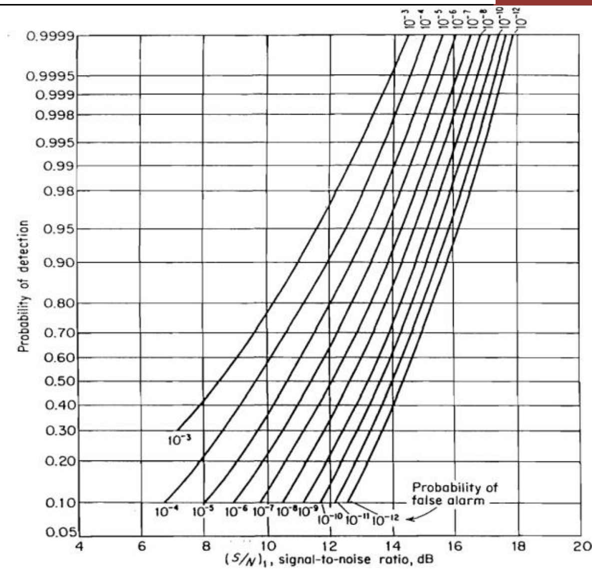



***Figure :*** Probability of detection for a sine wave in noise as a function of the signal- to-noise (power) ratio and the probability of false alarm.

For example, suppose that the desired false-alarm time was 15 min and the IF bandwidth was **1** M Hz. This gives a false-alarm probability of 1.11 x $10^{-9}$**.** Figure 2.7 indicates that a signal-to-noise ratio of **13.1** dB is required to yield a 0.50 probability of detection, 14.7 dB for 0.90, and 16.5 dB for 0.999.

## Radar Cross Section:

The size and ability of a target to reflect radar energy can be summarized into a single term, **σ**, known as the radar cross section RCS, which has units of $m^2$. If absolutely all of the incident radar energy on the target were reflected equally in all directions, then the radar cross section would be equal to the target's cross-sectional area as seen by the transmitter. In practice, some energy is absorbed and the reflected energy is not distributed equally in all directions. Therefore, the radar cross-section is quite difficult to estimate and is normally determined by measurement.

The target radar cross sectional area depends of:

- ❖ The airplane’s physical geometry and exterior features,
- ❖ The direction of the illuminating radar,
- ❖ The radar transmitters frequency,
- ❖ The used material types of the reflecting surface.

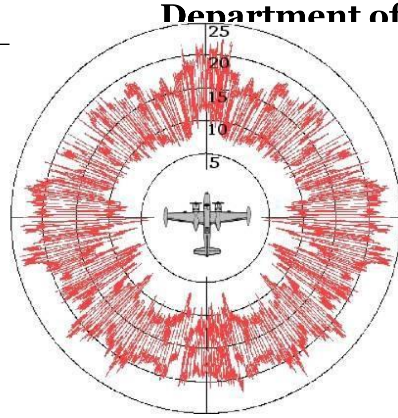



***Figure***: *T*he experimental radar cross section of a typical aircraft at 3 GHz frequency as a function of azimuth angle

| **Targets** | **RCS** [Sq m] |
|---|---|
| Jumbo Jet | 100 |
| jet airliner 13 … 16 | 20 to 40 |
| large fighter | 6 to 8 |
| helicopter | 3 to 7 |
| four-passenger jet | 2 to 3 |
| small aircraft | 1 |
| stealth jet | 0.1 |

**Table :** Examples of Radar Cross Section

## *Radar Cross Section of the target:*

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Or in other terms

$$\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density}/4\pi}$$

$$= \lim_{R\to\infty} 4\pi R^2 \left|\frac{E_r}{E_i}\right|^2$$

Where **R**= distance between radar and target

**$E_r$**= strength of reflected field at radar

**$E_i$**= strength of incident field at target

For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger will be the cross section.

***Scattering*** and ***diffraction:*** are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fields are the same, and one may talk about scattering and diffraction interchangeably.

**Radar cross section of a simple sphere**: is shown in the figure below as a function of its circumference measured in wavelengths.(**2πa/λ** where ***a*** is the radius of the sphere and λ is the wavelength). The plot consists of three regions.

***1. Rayleigh Region:***

- The region where the size of the sphere is small compared with the wavelength ≪ (**2πa/λ 1)** is called the **Rayleigh** region.
- The Rayleigh scattering region is of interest to the radar engineer because the cross sections of raindrops and other meteorological particles fall within this region at the usual radar frequencies.

***2. Optical region:***

- It is at the other extreme from the **Rayleigh** region where the dimensions of the sphere are large compared with the wavelength **(2πa/λ 1).** For large **2πa/λ**, the radar cross section approaches the optical cross section **$\pi a^2$**.

*3. Mie or Resonance region:*

- Between the optical and the Rayleigh region is the ***Mie,*** or resonance, region. The cross section is oscillatory with frequency within this region. The maximum value is 5.6 dB greater than the optical value, while the value of the first null is 5.5 dB below the optical value. (The theoretical values of the maxima and minima may vary according to the method of calculation employed.

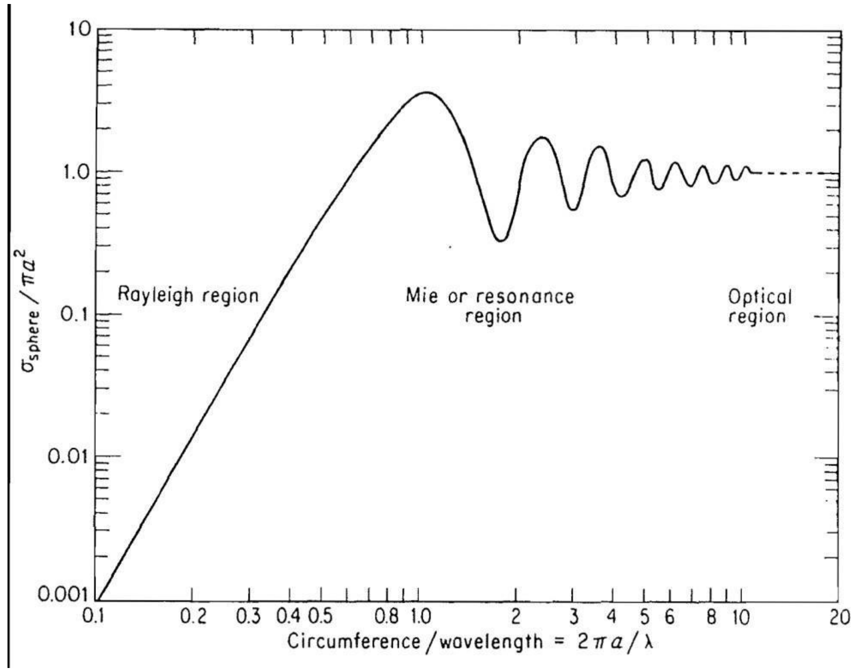



***Figure: Radar cross section of the sphere. a = radius; λ= wavelength.***

*Since the sphere is a sphere no matter from what aspect it is viewed, its cross section will not be aspect- sensitive. The cross section of other objects, however, will depend upon the direction as viewed by the radar. (Aspect angle)*

## Radar cross section of a cone-sphere:

An interesting radar scattering object is the cone-sphere, a cone whose base is capped with a sphere such that the first derivatives of the contours of the cone and sphere are equal at the joint. Figure below is a plot of the nose-on radar cross section. The cross section of the cone- sphere from the vicinity of the nose-on direction is quite low.

- Scattering from any object occurs from discontinuities. The discontinuities, and hence the backscattering, of the cone-sphere are from the tip and from the join between the cone and the sphere.
- The nose-on radar cross section is small and decreases as the square of the

wavelength. The cross section is small over a relatively large angular region. **A** large **specular***(having qualities of a mirror)*return is obtained when the cone-sphere is viewed at near perpendicular incidence to the cone surface, i.e., when $\theta = 90 - \alpha$, where $\alpha$= cone half angle. From the rear half of the cone- sphere, the radar cross section is approximately that of the sphere.

❖ The nose-on cross section of the cone-sphere varies, but its maximum value is approximately $\mathbf{0.4\lambda^2}$ and its minimum is $\mathbf{0.01\lambda^2}$ for a wide range of half-angles for frequencies above the Rayleigh region. The null spacing is also relatively insensitive to the cone half-angle.

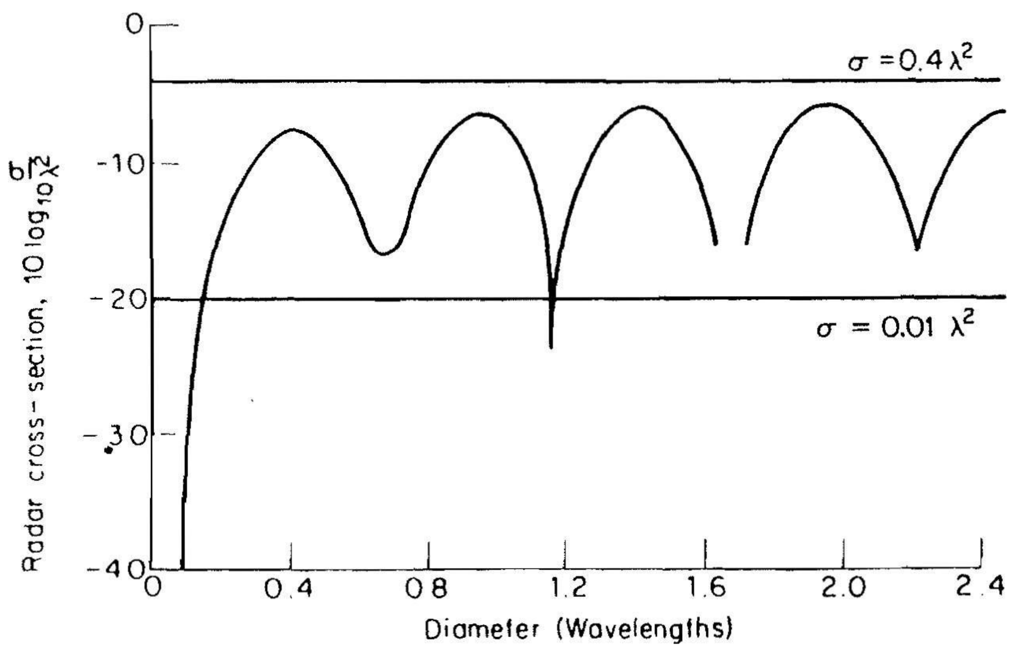



***Figure: Radar cross section of a cone sphere with 15$^0$ half angle as a function of the diameter in Wave lengths.***

In order to realize in practice the very low theoretical values of the radar cross section for a cone sphere, the tip of the cone must be sharp and not rounded, the surface must be smooth (roughness small compared to a wavelength), the join between the cone and the sphere must have a continuous first derivative, and there must be no holes, windows, or protuberances on the surface.

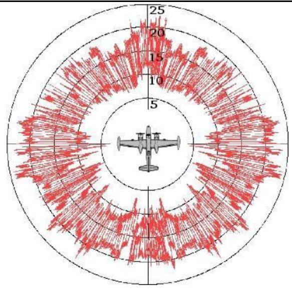
Shaping of the target, as with the cone-sphere, is a good method for reducing the radar cross section. Materials such as carbon-fiber composites, which are sometimes used in aerospace applications, can further reduce the radar cross section of targets as compared with that produced by highly reflecting metallic materials.

### *Complex targets:*

The radar cross section of complex targets such as ships, aircraft, cities, and terrain are complicated functions of the viewing aspect and the radar frequency. Target cross sections may be computed with the aid of digital computers, or they may be measured experimentally. The target cross section can be measured with full-scale targets, but it is more convenient to make cross-section measurements on scale models at the proper scaled frequency.

A complex target may be considered as comprising a large number of independent objects that scatter energy in all directions. The energy scattered in the direction of the radar is of prime interest. The relative phases and amplitudes of the echo signals from the individual scattering objects as measured at the radar receiver determine the total cross section. The phases and amplitudes of the individual signals might add to give a large total cross section, or the relationships with one another might result in total cancellation. In general, the behavior is somewhere between total reinforcement and total cancellation. If the separation between the individual scattering objects is large compared with the wavelength-and this is usually true for most radar applications-the phases of the individual signals at the radar receiver will vary as the viewing aspect is changed and cause a scintillating echo.

The most realistic method for obtaining the radar cross section of aircraft is to measure the actual target in flight. There is no question about the authenticity of the target being measured. An example of such a facility is the dynamic radar cross-section range of the U.S. Naval Research Laboratory. Radars at L, S, C and X bands illuminate the aircraft target in flight. The radar track data is used to establish the aspect angle of the target with respect to the radar. Pulse-to-pulse radar cross section is available, but for convenience in presenting the data the values plotted usually are an average of a large number of values taken within a $\mathbf{10^0}$ by $\mathbf{10^0}$ aspect angle interval. Examples of such data are given in Figure below.



It can be seen that the radar cross section of an aircraft is difficult to specify concisely. Slight changes in viewing aspect or frequency result in large fluctuations in cross section. Nevertheless, a single value of cross section is sometimes given for specific aircraft targets for use in computing the radar equation. There is no standard, agreed-upon method for specifying the single-valued cross section of an aircraft. The average value or the median might be taken. Sometimes it is a " minimum" value, perhaps the value exceeded 99 percent of the time or 95 percent of the time. It might also be the value which when substituted into the radar equation assures that the computed range agrees with the experimentally measured range.

## Radar Cross Section (RCS) fluctuations:

The fluctuations in the Radar Cross Section takes place due to any of the following mechanisms

1. **Variation in the viewing angle**: The target can be considered to be made of a number of small elemental scatterers. The RCS is the vector sum of the contribution of all elemental scatterers and depends on the angle at which the radar views them. The viewing angle in the horizontal plane is known as the target's aspect angle and I n the vertical plane it is known as the tilt angle. As the aspect angle changes, the relative distances from the elemental scatterers change causing different vector summations at the radar and results in variation in RCS. The degree of variation changes upon the target to target depending upon its complexity.
2. **Variation with frequency**: As the frequency of transmission varies, the round trip wavelengths of the scatterers vary and resulting vector sum varies. The spacing of the scatterers and the wavelength determines the amount of fluctuation for a given frequency change.
3. **Fluctuations from multipath:** There could be multiple paths of signal propagation in either direction. The echo received at the radar is a summation of the signals arriving through multiple paths depending upon the phase. As the range varies the multiple paths undergo a change and results in variation of RCS.

However, to properly account for target cross-section fluctuations, the probability density function and the correlation properties with time must be known for the particular target and type of trajectory. Curves of cross section as a function of aspect and a knowledge of the trajectory with respect to the radar are needed to obtain a true description of the dynamical variations of cross section. The probability-density function gives the probability of finding any particular value of target cross section between the values of σ and σ + dσ, while the autocorrelation function describes the degree of correlation of the cross section with time or number of pulses. It is usually not practical to obtain the experimental data necessary to compute the probability density function and the autocorrelation function from which the overall radar performance is determined. Most radar situations are of too complex a nature to warrant obtaining complete data. **A** more economical method to assess the effects of a fluctuating cross section is to postulate a reasonable model for the fluctuations and to analyze it mathematically.

***Swerling*** has calculated the detection probabilities for four different fluctuation models of cross section. In two of the four cases, it is assumed that the fluctuations are completely correlated during a particular scan but are completely uncorrelated from scan to scan. In the other two cases, the fluctuations are assumed to be more rapid and uncorrelated pulse to pulse. The four fluctuations models are as givenbelow.

## RCS Fluctuation Models:

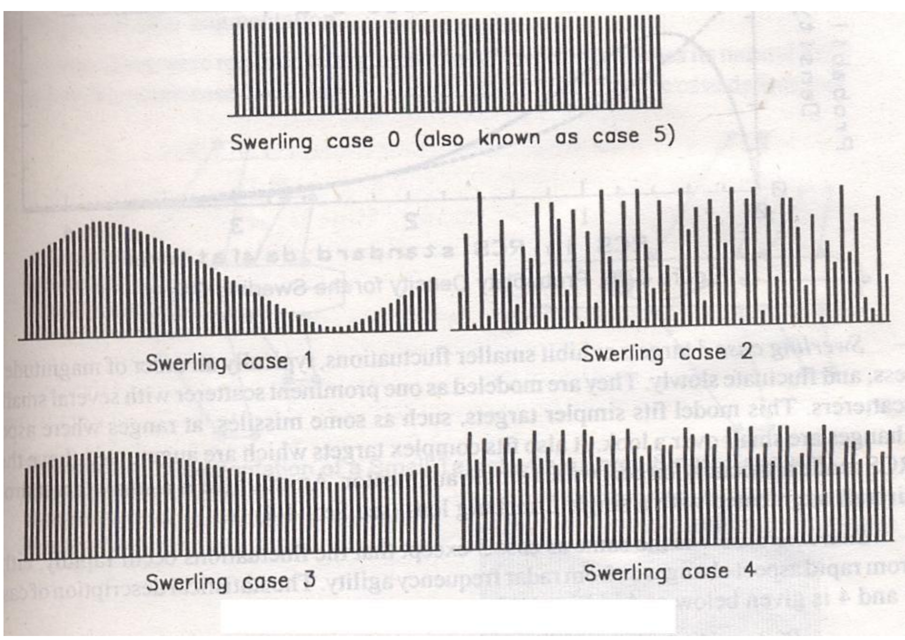



***Figure:*** Swerling Cases

The most commonly use models which help us in describing the behavior of a wide variety of targets are the **'Swerling models'.** The models are useful in calculation of Signal to Noise Ration for given probability of detection and False Alarm Rate.

1. **Swerling Case 0:** This is the model in which the targets are simple isotropic scatterers and RCS does not vary from pulse to pulse in the interval of our interest.
2. **Swerling Case 1:** Targets exhibit large fluctuations but occurring slowly. They are modeled as several scatterers of approximately equal to the RCS. Case 1 can be applied to complex targets such as aircraft.
3. **Swerling Case 2:** Targets exhibit large fluctuations but occurring rapidly. Case 1 can be applied to complex targets such as aircraft where aspect ratio is changing rapidly. It also fits complex targets for radar with pulse-to-pulse frequency agility.
4. **Swerling Case 3:** Target exhibit smaller fluctuations and fluctuate slowly. This model fits in simple targets such as missile.
5. **Swerling Case 4:** Target exhibit smaller fluctuations and fluctuate rapidly. This model fits in simple targets such as missile.

The Probability density function in respect of Case 1 and Case 2 targets is given by

$$P(\sigma) = (1/\mu)\, e^{-\sigma/\mu}$$

Where $P(\sigma)$ = Probability density function of a certain RCS

$\sigma$ = The RCS

$\mu$ = The median RCS

The Probability density function in respect of Case 3 and Case 4 targets is given by $P(\sigma) = (4\sigma/\mu^2)\, e^{-2\sigma/\mu}$

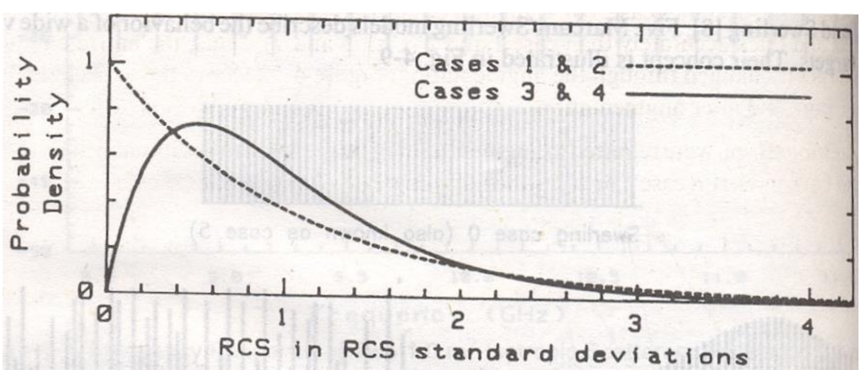



**Figure:** Probability Density for Swerling Cases

## Solved Examples:

1. Calculate the a maximum range of a radar system which operates at 3 cm with a peak pulse power of 500 kW, if its minimum receivable power is $10^{-13}$ W, the capture area of its antenna is 5 $m^2$ and the radar cross sectional area of the target is 20 Sq m.

Given

$\lambda = 3$ cm

$P_t = 500$ kW

$S_{min} = 10^{-13}$

$A_e = 5$ Sq m

$\sigma = 20$ Sq m

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}$$

$$G = \frac{4\pi A_e}{\lambda^2}$$

.

G = ( 4π x 5) / 0.0009 = 69841

$R_{max}^4$ = [ 500 x $10^3$ x 69841x 5 x 20] / [4π x 4π x$10^{-13}$] =

= [34920.6X $10^8$] / [ 158 X $10^{-13}$]

= 22101000 x $10^{16}$

$R_{max}$ = 685.6 km

2. Pulsed radar operating at 10 GHz has an antenna with a gain of 28 dB and a transmitter power of 2 kW. What is the maximum range of the radar if its defined to detect a target with a cross section of 12 Sq m and the minimum detectable signal is - 90dBm.

**Given**

**f = 10 G Hz., λ = 3 cm**

**G = 28 dB or G = 10 $^{2.8}$ = 630.9**

$P_t$ = 2 kW

S min = -90 dBm 0r $10^{-9}$

$$R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4}$$

σ = 12 Sq m

Rmax $^4$ = [ 2 x $10^3$ x 398034 x .0009 x 12] / [4π x 4π x4π x$10^{-9}$] =

= [8597.55 x $10^3$ ] / [1.986 x 10-$^6$ ]

= 432.9 x $10^{10}$

Rmax = 1442 m

3. Consider for a given radar, if minimum receiver sensitivity is -120dB, trans-mitted peak power is 100kW, gain of antenna is 30dB, target cross section is 5 square meter and maximum range of the radar is 300km, calculate the effective area of the receiving antenna.

S min = -120 dBm 0r $10^{-12}$

kW

G = 30 dB or G = 10 $^3$ = 1000

σ = 5 Sq m

Rmax =300 km

$P_t$ = 100

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}$$ = 2555 Sq m

4. A low power, short range radar is solid-state throughout, including a low-noise RF amplifier which gives it an overall noise figure of 4.77dB. If the antenna diameter is 1m, the IF bandwidth is 500kHz, the operating frequency is 8 GHz and the radar set is supposed to be capable of detecting targets of 5m$^2$ cross sectional area at a maximum distance of 12 km, what must be the peak transmitted pulse power?

## Objective type Questions:

1. The reflected signal from the target is called

   A. Clutter  **B. Echo**

   C. Noise  D. None of the above

2. The letter 'D' in acronym RADAR stands for

   A. Doppler  B. Duplexer

   **C. Detection**  D. Distance

3. The reflected signal from static objects is called

   A. Noise  B. Video Pulse

   C. Echo  **D. Clutter**

4. The unambiguous range of a Pulse Radar primarily depends upon

   A. Pulse Width  B. Frequency of RF signal

   **C. PRF**  D. Transmitted power

5. The range of radar is primarily restricted by

   **A. Line of sight**  B. Transmitted power

   C. Frequency of RF signal  D. Antenna directivity

6. The radar range is not influenced by

   A. Cross section of the target  B. PRF

   C. Antenna directivity  **D. Pulse width**

7. If 'T' is the transit time, the Range is given by

   A. T* c  **B. T*c/2**

   C. T/c  D. T/2c

8. The weakest signal that can be detected by radar receiver is called

A. Minimum target cross section **B. Minimum detectable signal**

C. Maximum detectable signal D. Minimum S/N ratio

9. Reduction of PRT below maximum unambiguous range value results in

A. Increase in false alarm probability B. **Multiple time around echoes**

C. Reduction in S/N ratio D. Enhancement of range resolution

10. If the maximum transmitted power of a radar in increased by 16 times with all other parameters unchanged, the radar range in increased by

A. 16 times B. 4 times

**C. 2 times** D. 8 times

11. The range resolution of a pulse radar depends upon

A. PRF B. **Pulse width**

C. Transmitted power D. Receiver sensitivity

12. The thermal noise power generated at the input of a receiver of band width '$B_n$' at a temperature 'T' (in Kelvin) is

A. $kT/ B_n$ B. $kT B_n^2$

C. $k B_n/T$ **D. $kT B_n$**

13. Reduction of PRT below maximum unambiguous range value results in

A. Increase in false alarm probability B. **Multiple time around echoes**

C. Reduction in S/N ratio D. Enhancement of range resolution

# Unit - 02

## CW Radar:

***Introduction:***

The Radar transmitter may be operated continuously rather than pulsed if the strong transmitted signal can be separated from the weak echo. The received-echo-signal power is considerably smaller than the transmitter power; it might be as little as $\mathbf{10}^{-18}$ that of the transmitted power-sometimes even less. Separate antennas for transmission and reception help segregate the weak echo from the strong leakage signal, but the isolation is usually not sufficient. **A** feasible technique for separating the received signal from the transmitted signal when there is relative motion between radar and target is based on recognizing the change in the echo-signal frequency caused by the doppler effect.

## Doppler Shift:

Doppler is the apparent change in wavelength (or frequency) of an electromagnetic or acoustic wave when there is relative movement between the transmitter (or frequency source) and the receiver.

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the **Doppler effect** and is the basis of CW radar.

If R is the distance from the radar to target, tile total number of wavelengths **λ** contained in the two-way path between the radar and the target is **2R/ λ**. The distance R and the wavelength **λ** are assumed to be measured in the same units. Since one wavelength corresponds to an angular excursion of **2π** radians, the total angular excursion **Φ** made by the electromagnetic wave during its transit to and from the target is **4πR/ λ** radians. If the target is in motion, R and the phase **Φ** are continually changing. **A** change in **4** with respect to time is equal to a frequency. This is the doppler angular frequency

$$\Phi = 2 \text{ x } 2\pi \text{ X } R/\lambda = 4\pi R/\lambda$$

Where **Φ** is total phase shift of the signal during transit time

$$\omega_d = 2\pi f_d = d\Phi/dt = 4\pi/\lambda \, dR/dt = (4\pi/\lambda) V_r$$

Where '$\omega_d$' is Doppler angular frequency and '$f_d$'is the Doppler frequency $v_r$ is the relative radial velocity of the target

$$f_d = 2 V_r / \lambda$$

The relative velocity may be written $V_r$ = V cos θ, where 'V' is the target speed and 'θ' is the angle made by the target trajectory and the line joining radar and target. When θ = 0 the doppler frequency is maximum. The doppler is zero when the trajectory is perpendicular to the radar line of sight i.e. $\theta = 90^0$

The CW radar which employs a continuous transmission, either modulated or un-modulated is of interest not only because of its many applications, but its study also serves as a means for better understanding the nature and use of the doppler information contained in the echo signal, whether in a CW or a pulse radar (MTl) application. In addition to allowing the received signal to be separated from the transmitted signal, the CW radar provides a measurement of relative velocity which may be used to distinguish moving targets from stationary objects or clutter.

## CW Radar:

Consider the simple CW radar as illustrated by the block diagram**.** The transmitter generates a continuous (un-modulated) oscillation of frequency $f_o$, which is radiated by the antenna through a circulator. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna. If the target is in motion with a velocity $V_r$ **,** relative to the radar, the received signal will be shifted in frequency from the transmitted frequency $f_o$ by an amount $\pm f_d$ as given. The plus sign associated with the doppler frequency applies if the distance between target and radar is decreasing (closing target), that is, when the received signal frequency is greater than the transmitted signal frequency.

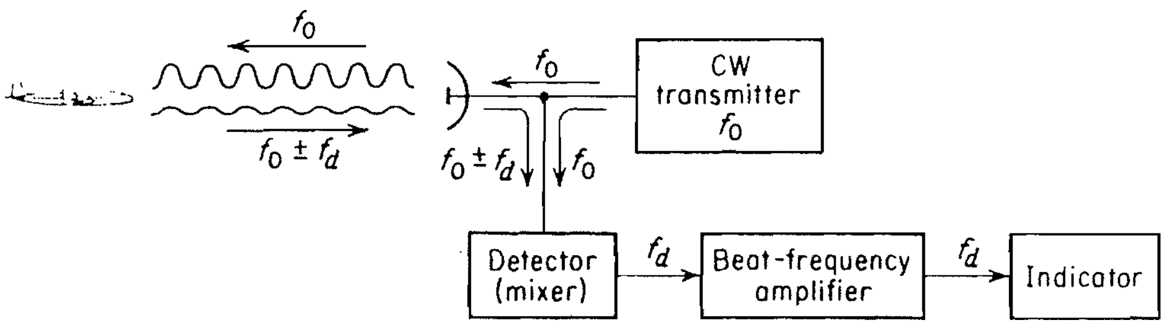



**Figure:** Simple CW radar block diagram

The minus sign applies if the distance is increasing (receding target). The received echo signal at a frequency f± $f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal $f_o$ to produce a doppler beat note of frequency $f_d$. The sign of. $f_d$ is lost in this process.

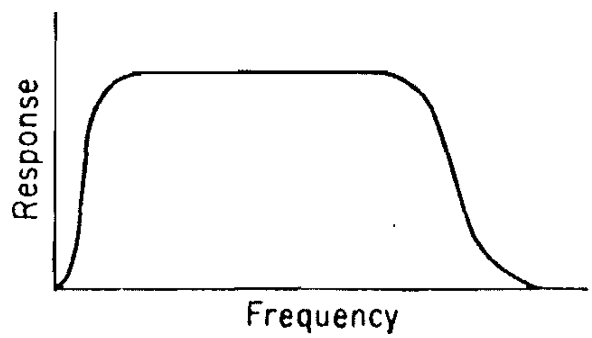



**Figure:** Response characteristic of beat-frequency amplifier.

The purpose of the doppler amplifier is to eliminate echoes from stationary targets and to amplify the doppler echo signal to a level where it can operate an indicating device. It might have a frequency-response characteristic similar to that of above Figure. The low-frequency cutoff must be high enough to reject tile d-c component caused by stationary targets, but yet it must be low enough to pass the smallest doppler frequency expected.

Sometimes both conditions cannot he met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest doppler frequency expected. The design of Doppler frequency amplifier is a challenge to design engineers.

## Isolation between Transmitter and Receiver:

A single antenna serves the purpose of transmission and reception in the simple CW radar described above. In principle, a single antenna may be employed since the necessary isolation between the transmitted and the received signals is achieved via separation in frequency as a result of the doppler effect. In practice, it is not possible to eliminate completely the transmitter leakage.

There are two practical effects which limit the amount of transmitter leakage power which can be tolerated at the receiver. These are

(1) The maximum amount of power the receiver input circuitry can withstand before it is physically damaged or its sensitivity reduced (burnout) and

(2) The amount of transmitter noise due to hum, microphonics, stray pick-up, and instability which enters the receiver from the transmitter.

The additional noise introduced by the transmitter reduces the receiver sensitivity. Except where the CW radar operates with relatively low transmitter power and insensitive receivers, additional isolation is usually required between the transmitter and the receiver if tile sensitivity is not to be degraded either by burnout or by excessive noise.

The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and the sensitivity of the receiver. For example, if the safe value of power which might be applied to a receiver were 10 mW and if the transmitter power were 1 kW, the isolation between transmitter and receiver must be at least 50 dB.

It will be recalled from previous chapter that the receiver of a pulsed radar is isolated and protected from the damaging effects of the transmitted pulse by the duplexer, which short- circuits the receiver input during the transmission period. Turning off the receiver during transmission with a duplexer is not possible in a CW radar since the transmitter is operated continuously.

**Isolation between transmitter and receiver might be obtained with a single antenna by using a hybrid junction, circulator, turnstile junction, or with separate polarizations.** Separate antennas for transmitting and receiving might also be used. The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB. In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. One limitation of the hybrid junction is the 6-dB loss in overall performance which results from the inherent waste of half the transmitted power and half the received signal power.

Ferrite isolation devices such as the circulator do not suffer the 6-dB loss inherent in the hybrid junction. Practical devices have isolation of the order of 20 to 50 dB.

## Non-Zero Intermediate-frequency Receiver:

The receiver of the simple CW radar is in some respects analogous to a super- heterodyne receiver. Receivers of this type are called homodyne receivers, or super- heterodyne receivers with zero IF. The function of the local oscillator is replaced by the leakage signal from the transmitter. Such a receiver is simpler than one with a more conventional intermediate frequency since no IF amplifier or local oscillator is required.

However, the simpler receiver is not as sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect. Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the **flicker effect varies as 1/f.** This is in contrast to shot noise or thermal noise, which is independent of frequency. Thus, at the lower range of frequencies (audio or video region), where the doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity. For short-range, low-power, applications this decrease in sensitivity might be tolerated since it can be compensate by a modest increase in antenna aperture and/or additional transmitter power. But for maximum efficiency with CW radar, the reduction in the sensitivity caused by the simple Doppler receiver with zero IF, cannot be tolerated.

The effects of flicker noise are overcome in the normal super-heterodyne receiver by using an intermediate frequency, high enough to render the flicker noise small compared with the normal receiver noise. This results from the inverse, frequency dependence of flicker noise. Above figure shows a block diagram of the CW radar whose receiver operates with a nonzero IF**.** Separate antennas are shown for transmission and reception instead of the usual local oscillator found in the conventional super-heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.

Since the output of the mixer consists of two sidebands on either side of the carrier plus higher harmonics, a narrowband filter selects one of the sidebands as the reference signal. The improvement in receiver sensitivity with an intermediate-frequency super-heterodyne might be as much as 30 dB over the simple receiver.

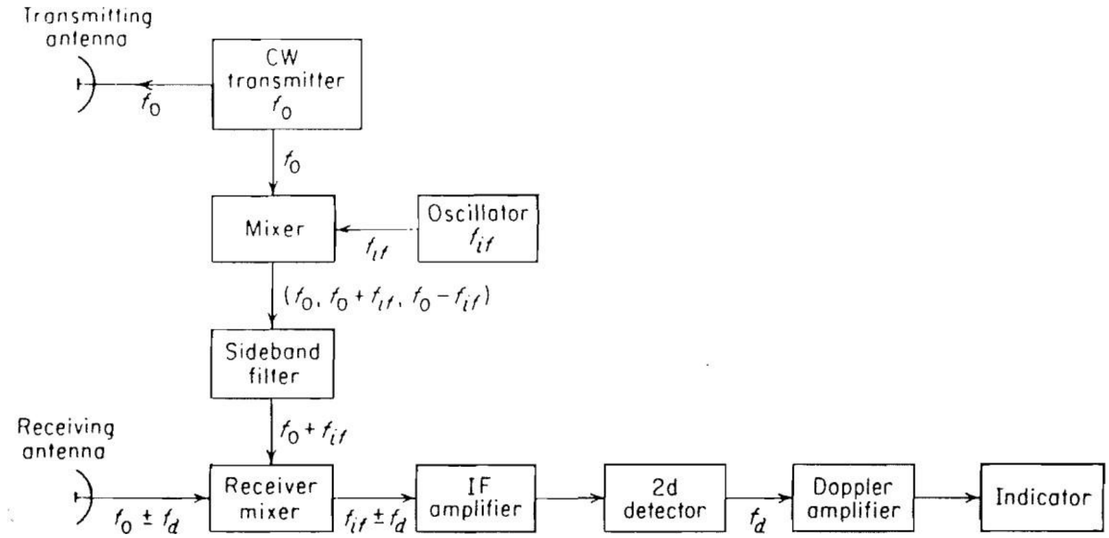



**Figure:** Block diagram of CW doppler radar with nonzero IF receiver,

## Receiver Bandwidth:

One of the requirements of the doppler-frequency amplifier in the simple CW radar or the IF amplifier of the sideband super-heterodyne is that it be wide enough to pass the expected range of doppler frequencies. In most cases of practical interest the expected range of doppler frequencies will be much wider than the frequency spectrum occupied by the signal energy. Consequently, the use of a wideband amplifier covering the expected doppler range will result in an increase in noise and a lowering of the receiver sensitivity. If the frequency of the doppler-shifted echo signal were known beforehand, a narrowband filter-one just wide enough to reduce the excess noise without eliminating a significant amount of signal energy-might be used. If the waveform of the echo signal were known, as well as its carrier frequency, the matched filter could be specified, which will be discussed at depth in subsequent chapters.

Several factors tend to spread the CW signal energy over a finite frequency band. These must be known if an approximation to the bandwidth required for the narrowband Doppler filter is to be obtained.

If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function and the receiver bandwidth would be infinitesimal. But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The rnore normal situation is an echo signal which is a sine wave of finite rather than infinite duration. The frequency spectrum of a finite-duration sine wave has a shape of the form $[\sin \pi(f-f_o)\delta]/\pi(f-f_o))$, where $f_o$ and $\delta$ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted. Practical receivers can only approximate this characteristic. (Note that this is the same as the Spectrum of a pulse of sine wave, the only difference being the relative value of the duration $\delta$) In many instances, the echo is not a pure sine wave of finite duration but is perturbed by fluctuations in cross section, target accelerations, scanning fluctuations, etc., which tend to broaden the bandwidth still further. Some of these spectrum-broadening effects are considered below.

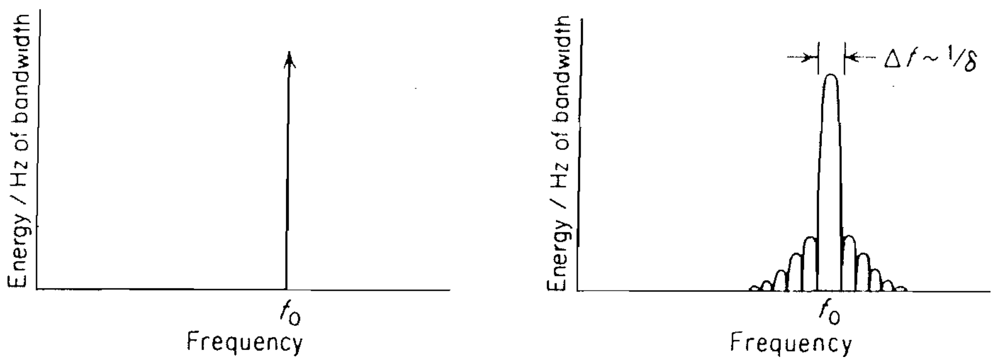



**Figure:** Frequency spectrum of CW oscillation of (a) infinite duration and ( b ) finite duration.

Assume a **CW** radar with an antenna beamwidth of $\theta_B$**,** deg scanning at the rate of $d\theta_S/dt$ deg/s. The time on target (duration of the received signal) is $\delta = \theta_B$,/( $d\theta_S/dt$ ). Thus the signal is of finite duration and the bandwidth of the receiver must be of the order of the reciprocal of the time on target $d\theta_S/dt$ /($\theta_B$).,. Although this is not an exact relation, it is a good enough approximation for purposes of the present discussion. If the antenna beamwidth were $2^0$ and if the scanning rate were $36^0$/s (6 rpm), the spread in the spectrum of the received signal due to the finite time on target would be equal to 18 Hz, independent of 'the transmitted frequency.

## Sign of the Radial Velocity:

In some applications of CW radar it is of interest to know whether the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency. If the echo-signal frequency lies below the carrier, the target is receding; if the echo frequency is greater than the carrier, the target is approaching. Although the doppler-frequency spectrum "folds over" in the video because of the action of the detector, it is possible to determine its sign from a technique borrowed from single-sideband communications. If the transmitter signal is given by

$$E_t = E_0 \cos \omega_o t$$

the echo signal from a moving target will be

$$E_r = k_1 E_0 \cos\{(\omega_o \pm \omega_d)t + \Phi\}$$

Where; $E_o$ = amplitude of transmitter signal

$k_1$ = a constant determined from the radar equation

$\omega_o$ = angular frequency of transmitter, rad/s

$\omega_d$ = dopper angular frequency shift

$\Phi$ = a constant phase shift, which depends upon range of initial detection

The sign of the doppler frequency, and therefore the direction of target motion, may be found I by splitting the received signal into two channels as shown in Figure. In channel **A** the signal is processed as in the simple CW radar. The received signal and a portion of the transmitter heterodyne in the detector (mixer) to yield a difference signal

$$E_A = k_2 E_0 \cos\{\pm \omega_d t + \Phi\}$$

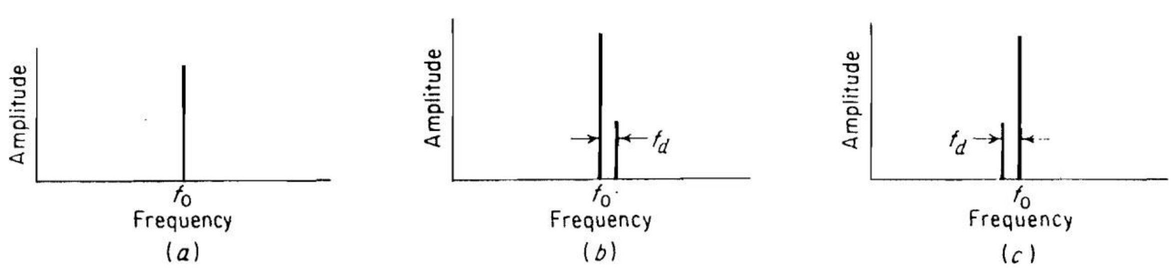



**Figure:** Spectra of received signals. (a) No doppler shift, no relative target motion; (b) approaching target; (c) receding target.

The other channel is similar, except for a $90^0$ phase delay introduced in the reference signal. The output of the channel B mixer is

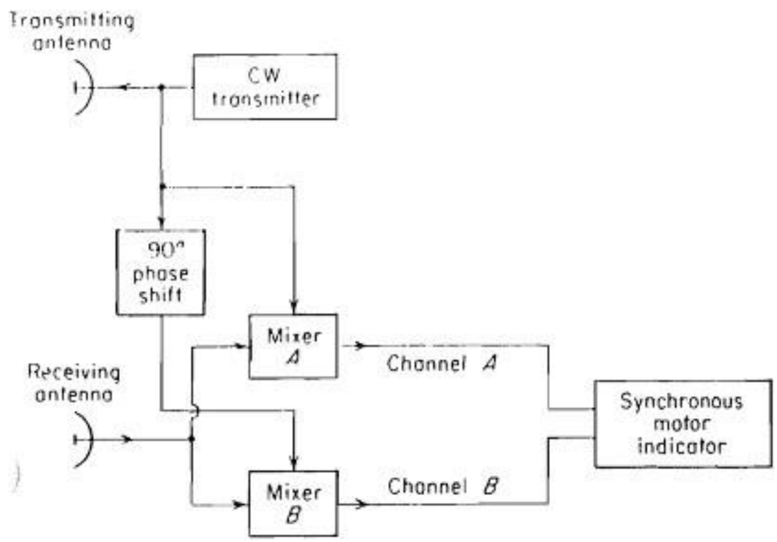



**Figure:** Measurement of doppler direction using synchronous, two-phase motor.

$$E_B = k_2 E_0 \cos \{\pm \omega_d t + \Phi + \pi/2\}$$

If the target is approaching (positive doppler) the outputs from the two channels are

$$E_A (+) = k_2 E_0 \cos \{ \omega_d t + \Phi \}$$

$$E_B (+) = k_2 E_0 \cos \{ \omega_d t + \Phi + \pi/2 \}$$

On the other hand, if the targets are receding (negative doppler),

$$E_A (-) = k_2 E_0 \cos \{ \omega_d t - \Phi \}$$

$$E_B (-) = k_2 E_0 \cos \{ \omega_d t - \Phi - \pi/2 \}$$

The sign of $\omega_d$ and the direction of the target's motion may be determined according to whether the output of channel **B** leads or lags the output of channel **A.** One method of determining the relative phase relationship between the two channels is to apply the outputs to a synchronous two-phase motor. The direction of motor rotation is an indication of the direction of the target motion. Electronic methods may be used instead of a synchronous motor to sense the relative phase of the two channels.

## Applications of CW Radar:

The chief use of the simple, un-modulated CW radar is for

(a) The measurement of the relative velocity of a moving target, as in the police speed monitor.
(b) Rate-of-climb meter for vertical-take-off aircraft.
(c) In support of automobile traffic, CW radar has been suggested for the control of traffic lights, regulation of toll booths, vehicle counting, as a replacement for the " fifth-wheel" speedometer in vehicle testing.
(d) As a sensor in antilock braking systems, and for collision avoidance for railways.
(e) CW radar can be used as a speedometer to replace the conventional axle-driven tachometer.
(f) CW radar is also employed for monitoring the docking speed of large ships.
(g) It has also seen application for intruder alarms and for the measurement of the velocity of missiles, ammunition, and baseballs.

One of the greatest shortcomings of the simple CW radar is its inability to obtain a measurement of range. This limitation can be overcome by modulating the CW carrier, as in the frequency-modulated radar described in the next section.

## FMCW Radar:

***Introduction:***

The inability of the simple CW radar to measure range is related to the relatively narrow spectrum (bandwidth) of its transmitted waveform. Some sort of timing mark must be applied to a CW carrier if range is to be measured. The timing mark permits the time of transmission and the time of return to be recognized. The sharper or more distinct the mark, the more accurate the measurement of the transit time. But the more distinct the timing mark, the broader will be the transmitted spectrum. This follows from the properties of the Fourier transform. Therefore a finite spectrum must of necessity be transmitted if transit time or range is to be measured.

The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude, frequency, or phase. An example of an amplitude modulation is the pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum. **A** widely used technique to broaden the spectrum of CW radar is to frequency-modulate the carrier. The timing mark is the changing frequency. The transit time is proportional to the difference in frequency between the echo signal and the transmitter signal. The greater the transmitter frequency deviation in a given time interval, the more accurate the measurement of the transit time and the greater will be the transmitted spectrum.

## Range and Doppler measurement:

In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time in a known manner. Assume that the transmitter frequency increases linearly with time, as shown by the solid line in Figure. If there is a reflecting object at a distance R, an echo signal will return after a time T = 2R/c. The dashed line in the figure represents the echo signal.

If the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note $\mathbf{f_b}$ will be produced. If there is no doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and $\mathbf{f_b} = \mathbf{f_r}$ where $\mathbf{f_r}$ is the beat frequency due only to the target's range. If the rate of change of the carrier frequency is $df_0/dt$ the beat frequency is

$$f_r = \dot{f}_0 T = \frac{2R}{c} \dot{f}_0$$

In any practical CW radar, the frequency cannot be continually changed in one direction only. Periodicity in the modulation is necessary, as in the triangular frequency -modulation waveform shown in Fig. 4.1b. The modulation need not necessarily be triangular; **it** can be saw tooth, sinusoidal, or some other shape. The resulting beat frequency as a function of **time** is shown in Figure for triangular modulation. The beat note is of constant frequency except at the turn-around region. If the frequency is modulated at a rate $\mathbf{f_m}$ over a range $\Delta f$ the beat frequency is

$$f_r = \frac{2R}{c} 2f_m$$

$$\Delta f = \frac{4Rf_m \Delta f}{c}$$

Thus the measurement of the beat frequency determines the range R.

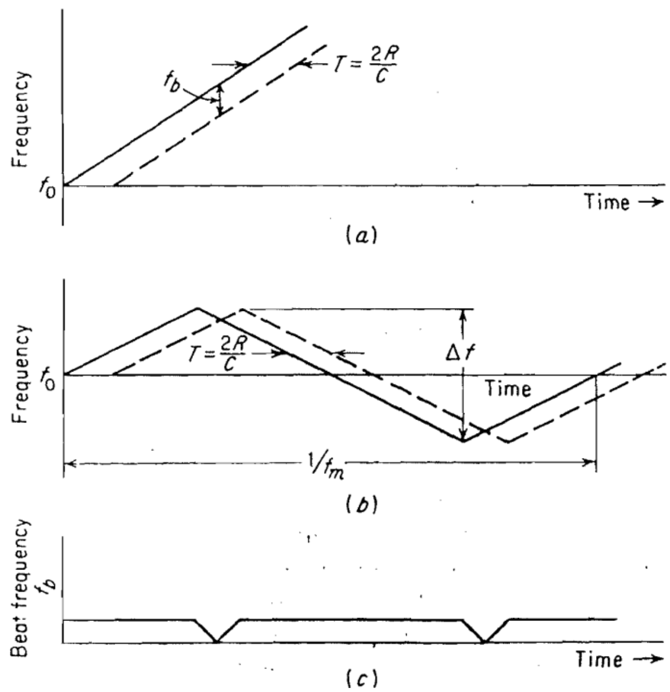



**Figure:** Frequency-time relation-ships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation; (b) triangular frequency modulation;(c) beat note of(b)

## Block diagram of FM-CW Radar:

**A** block diagram illustrating the principle of the FM-CW radar is shown in Figure below. A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

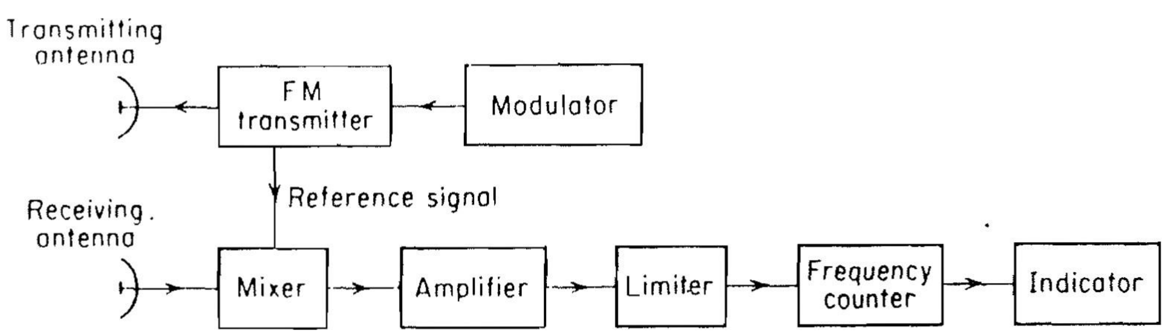



**Figure:** Block diagram of FM-CW radar.

Ideally. the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas. The beat frequency is amplified and limited to remove any amplitude fluctuations. The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated it in distance.

In the above, the target was assumed to be stationary. If this assumption is not applicable, a doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results. The doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down. On one portion of the frequency- modulation cycle. the beat frequency is increased by the doppler shift, while on the other portion, it is decreased. If for example, the target is approaching the radar, the beat frequency $\mathbf{f_b(up)}$ produced during the increasing, or up, portion of the FM cycle will be the difference between the beat frequency due to the range $\mathbf{f_r}$ and the doppler frequency shift $\mathbf{f_d}$ Similarly, on the decreasing portion, the beat frequency $\mathbf{f_b(down)}$ is the sum of the two.

$$\mathbf{f_b(up)= f_r + f_d}$$

$$\mathbf{f_b(down)= f_r - f_d}$$

The range frequency $\mathbf{f_r}$ may be extracted by measuring the average beat frequency; that is,

$$\mathbf{f_r = ½ [ f_b(up)+ f_b(down)]}$$

If $\mathbf{f_b(up}$ and $\mathbf{f_b(down)}$ are measured separately, for example, by switching a frequency counter every half modulation cycle, one-half the difference between the frequencies will yield the doppler frequency. This assumes $\mathbf{f_r} > \mathbf{f_d}$ . If, on the other hand, $\mathbf{f_r} < \mathbf{f_d}$, such as might occur with a high-speed target at short range, the roles of the averaging and the difference- frequency measurements are reversed; the averaging meter will measure Doppler velocity, and the difference meter, range. If it is not known that the roles of the meters are reversed because of a change in the inequality sign between $\mathbf{f_r}$ and $\mathbf{f_d}$, an incorrect Interpretation of the measurements may result.

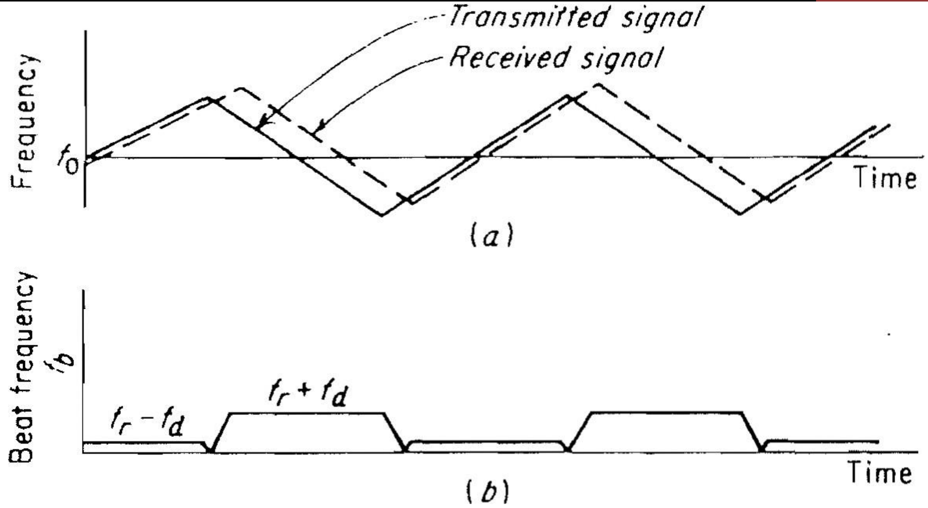



**Figure:** Frequency-time relationships in FM-CW radar when the received signal is shifted in frequency by the doppler effect

(a) Transmitted (solid curve) and echo (dashed curve) frequencies;

(b) beat frequency.

When more than one target is present within the view of the radar, the mixer output will contain more than one difference frequency. If the system is linear, there will be a frequency component corresponding to each target. In principle, the range to each target may be determined by measuring the individual frequency components and applying to each. To measure the individual frequencies, they must be separated from one another. This might he accomplished with a bank of narrowband filters, or alternatively, a single frequency corresponding to a single target may be singled out and continuously observed with a narrow band tunable filter. But if the motion of the targets were to produce a Doppler frequency shift, or if the frequency-modulation waveform were nonlinear, or if the mixer were not operated In its linear region, the problem of resolving targets and measuring the range of each becomes more complicated.

If the FM-CW radar is used for single targets only, such as in the radio altimeter, it is not necessary to employ a linear modulation waveform. This is certainly advantageous since a sinusoidal or almost sinusoidal frequency modulation is easier to obtain with practical equipments than are linear modulations. Any reasonable-shape modulation waveform can be used to measure the range, provided the average beat frequency is measured. If the target is in motion and the beat signal contains a component due to the -doppler frequency shift, the range frequency can be extracted, as before, if the average frequency is measured. To extract the doppler frequency, the modulation waveform must have equal upsweep and downsweep time intervals.

# MTI & PULSE DOPPLER RADAR:

***Introduction:***

The doppler frequency shift produced by a moving target may be used in a pulse radar just as in the CW radar**,**

- ❖ To determine the relative velocity of a target or
- ❖ To separate desired moving targets from undesired stationary objects (clutter).

Although there are applications of pulse radar where a determination of the target's relative Velocity is made from the doppler frequency shift, the use of doppler to separate small moving Targets in the presence of large clutter has probably been of far greater interest. Such a pulse Radar that utilizes the doppler frequency shift as a means for discriminating moving from fixed targets is called an **MTI** (moving target indication) or a **pulse doppler** radar. The two are based on the same physical principle, but in practice there are generally recognizable differences between them.

- ➢ The MTI radar, for instance, usually operates with ambiguous Doppler measurement (so-called **blind** speeds) but with unambiguous range measurement (no second- time'-around echoes).
- ➢ The opposite is generally the case for a pulse doppler radar. Its pulse repetition frequency is usually high enough to operate with unambiguous doppler (no blind speeds) but at the expense of range ambiguities.

## Description of Operation:

A CW radar may be converted into a pulse radar as shown in by providing a power amplifier and a modulator to turn the amplifier on and off for the purpose of generating pulses. The main difference between the pulse radar of below figure and the one described in Chap. 1 is that a small portion of the CW oscillator power that generates the transmitted pulses is diverted to the receiver to take the place of the local oscillator. However, this CW signal does more than function as a replacement for the local oscillator. It acts as the coherent reference needed to detect the doppler frequency shift. **By coherent it is meant that the phase of the transmitted signal is preserved in the reference signal. The reference signal is the distinguishing feature of coherent MTI radar.**

If the CW oscillator voltage is represented as $\mathbf{A_1 \sin 2\pi f_0 t}$, Where $\mathbf{A_1}$ is the amplitude and $\mathbf{f_0}$ is the carrier frequency.

Let the reference signal be written as **$A_2$ sin 2πf$_0$t**

The echo signal from a moving target can be written

$$V_{echo} = A_3 \sin\{2\pi(f_o \pm f_d)t - \frac{4\pi f_o R_o}{c}\} = A_3 \sin\{2\pi(f_o \pm f_d)t - \Phi_o\}$$

Where

**$A_2$** is the amplitude of reference signal

**$A_3$** is the amplitude of echo signal

**$f_d$** is the Doppler shift in frequency.

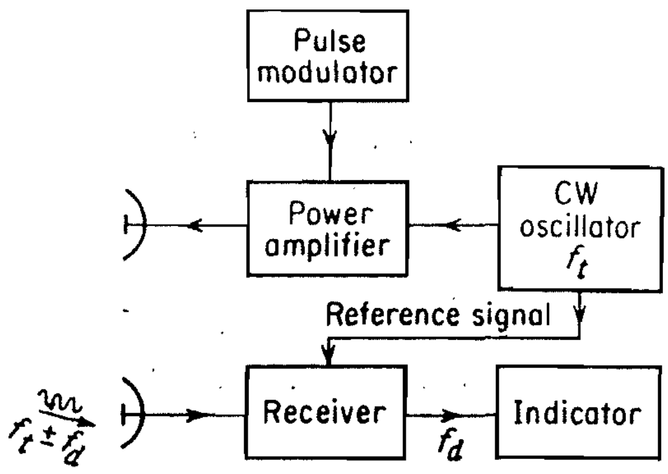



**Fig:** Simple Pulse Doppler radar

The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver. Only the low-frequency (difference-frequency) component from the mixer is of interest and is a voltage given by

$$V_{diff} = A_4 \sin\{2\pi f_d t - \frac{4\pi f_o R_o}{c}\}$$

**Note that for stationary targets the doppler frequency shift will be zero; hence Vdiff will not vary with time and may take on any constant value from +A4 to - A4 including zero. However, when the target is in motion relative to the radar, fd has a value other than zero and the Vdiff will be a function of time.**

Moving targets may be distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. range). **A** single sweep on an A-scope might appear as in Figure below.

This sweep shows several fixed targets and two moving targets indicated by the two arrows. On the basis of a single sweep, moving targets cannot be distinguished from fixed targets. Successive A-scope sweeps (pulse-repetition intervals) are shown in Figure below**.** Echoes from fixed targets remain constant throughout, but echoes from moving targets vary in amplitude from sweep to sweep at a rate corresponding to the doppler frequency. The superposition of the successive A-scope sweeps is shown in Figure. The moving targets produce, with time, a **"butterfly" effect** on the A-scope.

Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form suitable for display on the PPI scope is with a delay-line canceller.

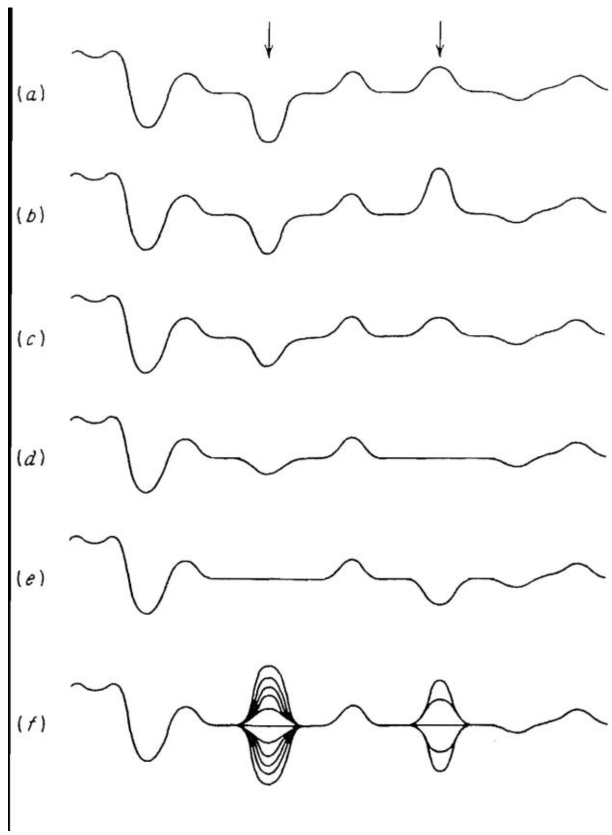



**Figure:** Successive sweeps of an MTI radar A-scope display (echo amplitude as a function of time); (f) superposition of many sweeps; arrows indicate position of moving targets.

The delay-line canceller acts as a filter to eliminate the dc component of fixed targets and to pass the ac components of moving targets. The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse-repetition frequency). The outputs from the two channels are subtracted from one another. The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse **to** pulse subtraction results in an uncancelled residue.

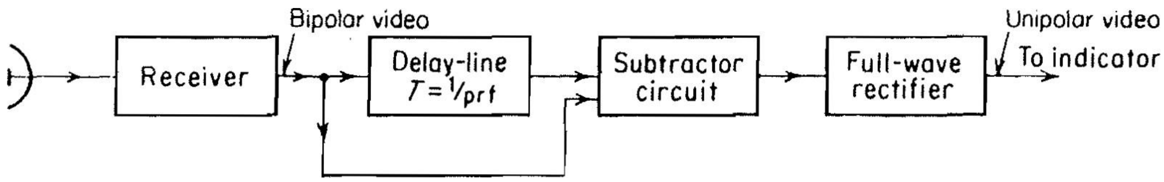



**Figure:** MTI receiver with delay-line canceller.

The output of the subtraction circuit is **bipolar video**, just as was the input. Before bipolar video can intensity-modulate a PPI display, it must be converted to unipotential voltages (unipolar video) by a full-wave rectifier.

### Remedy for Blind speeds:

The effect of blind speeds can be significantly reduced, without incurring range ambiguities, by

(a) Operating with more than one pulse repetition frequency. This is called a staggered-PRF MTI.

(b) Operating at more than one **RF** frequency can also reduce the effect of blind speeds.

## Digital Signal Processing:

The introduction of practical and economical digital processing to MTI radar allowed a significant increase in the options open to the signal processing designer. The convenience of digital processing means that the delay-line cancellers with tailored frequency-response characteristics can he readily achieved. A digital MTI does not, in principle, do any better than a we;; designed analog canceller; but it is more dependable, it requires less adjustments and attention, and can do some tasks easier. Most of the advantages of a digital MTI processor are due to its use of digital delay line rather than analog delay lines which are characterized by variations due to temperature, critical gains, and poor on-line availability.

**A** simple block diagram of a digital MTI processor is shown in Figure below. From the output of the IF amplifier the signal is split into two channels. One is denoted **I,** for **in-phase channel.** The other is denoted **Q,** for **quadrature** channel, since a $90^0$ phase change ($\pi/2$ radians) is two detectors to be

$90^0$ out of phase. The purpose of the quadrature channel is to eliminate the effects of blind **phases,** as will be described later. It is desirable to eliminate blind phases in any MTI processor, but it is seldom done with analog delay-line cancellers because of the complexity of the added analog delay lines of the second channel. The convenience of digital processing allows the quadrature channel to be added without significant burden so that it is often included in digital processing systems. It is for this reason it is shown in this block diagram, but was not included in the previous discussion of **MTI** delay-line cancellers.

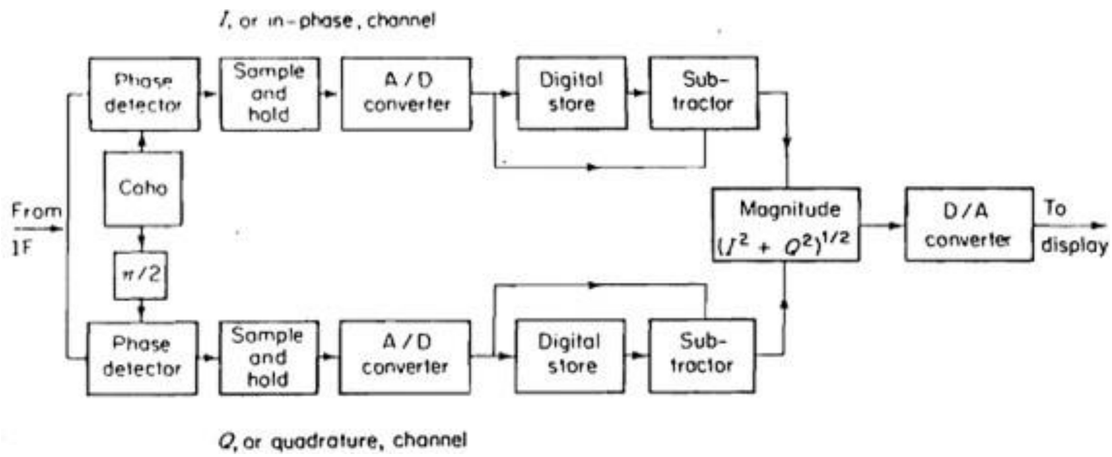



**Figure:** Block diagram of a simple digital MTI signal processor.

Following the phase detector the bipolar video signal is sampled at a rate sufficient to obtain one or more samples within each range resolution cell. These voltage samples are converted to a series of digital words by the analog-to-digital **(A/D)** converter. The digital words are stored in a digital memory for one pulse repetition period and are then subtracted from the digital words of the next sweep. The digital outputs of the I and Q channels are combined by taking the square root of $\mathbf{I^2 + Q^2}$. An alternative method of combining, which is adequate for most cases, is to take III + I**Q**I. The combined output is then converted to an analog signal by the digital-to-analog **(D/A)** converter. The unipolar video output is then ready to be displayed. The digital MTI processor depicted in Fig. **4.14** is that of a single-delayline canceller. Digital processors are likely to employ more complex filtering schemes, but the simple canceller is shown here for convenience. Almost any type of digital storage device can be used. A shift register is the direct digital analogy of a delay line, but other digital computer memories can also be used effectively.

The **A/D** converter has been, in the past, one of the critical parts of the MTI signal processor. It must operate at a speed high enough to preserve the information content of the radar signal, and the number of bits into which it quantizes the signal must be sufficient for the precision required. The number of bits in the A/D converter determines the maximum **improvement factor** the MTI radar can achieve. Generally the A/D converter is designed to cover the peak excursion of the phase detector output. A limiter may be necessary to ensure this. An N-bit converter divides the output of the phase detector into $\mathbf{2^N}$ - 1 discrete intervals.

## Blind Phases:

In the above it was said that the addition of the Q channel removed the problem of reduced sensitivity due to blind **phases.**

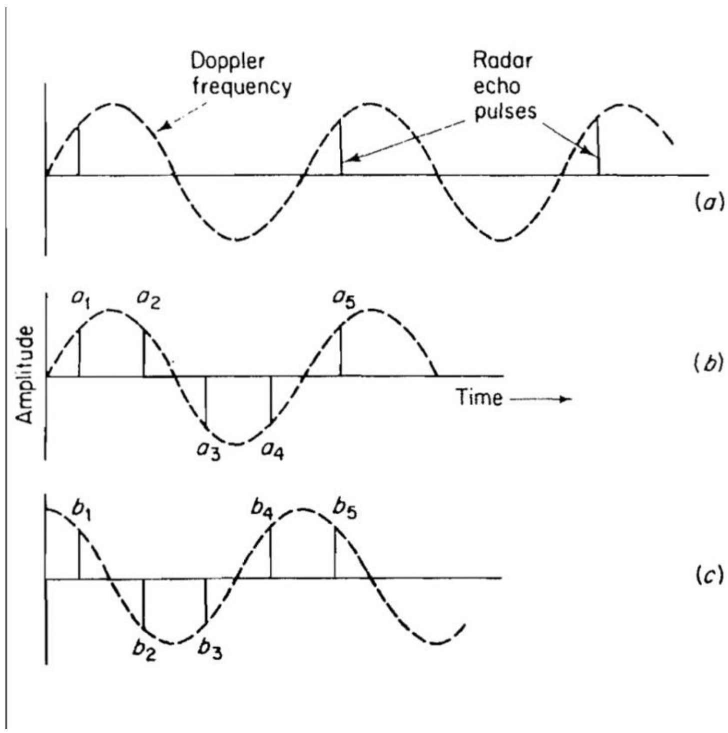



**Figure:** (a) Blind speed in an MTI radar. The target doppler frequency is equal to the prf. (b) Effect of blind phase in the I channel, and (c) in the Q channel.

This is different than the blind speeds which occur when the pulse sampling appears at the same point in the doppler cycle at each sampling instant. Above figure shows the in-phase, or I, channel with the pulse train such that the signals are of the same amplitude and with a spacing such that when pulse $a_1$ is subtracted from pulse $a_2$, the result is zero. However, a residue is produced when pulse $a_3$, is subtracted from pulse $a_4$, but not when **as** is subtracted from $a_4$, and so on.

In the quadrature channel, the doppler-frequency signal is shifted $90^0$ so that those pulse pairs that were lost in the **I** channel are recovered in the **Q** channel, arid vice versa. The combination of the I and Q channels thus results in a uniform signal with no loss. The phase of the pulse train relative to that of the doppler signal. With other phase arid frequency relationships, there is still a loss with a single channel MTI that can be recovered by the use of both the **I** and **Q** channels. An extreme case where the blind phase with only a single channel results in a complete loss of signal is when the doppler frequency is half the prf and the phase relationship between the two is such that the echo pulses lie on the zeros of the doppler-frequency sine wave. This is not the condition for a blind speed but nevertheless there is no signal. However, if the phase relationship is shifted $90^0$ as it is in the Q channel, then all the echo pulses occur at the peaks of the doppler-frequency sine wave. Thus, to ensure the signal will be obtained without loss, both **I** and Q channels are desired.

## Advantages of Digital Signal Processing:

Digital signal processing has some significant advantages over analog delay lines, particularly those that use acoustic devices.

- ❖ It is possible to achieve greater stability, repeatability, and precision with digital processing than with analog delayline cancellers.
- ❖ The reliability is better.
- ❖ No special temperature control is required, and it can be packaged in convenient size.
- ❖ The dynamic range is greater since digital MTI processors do not experience the spurious responses which limit signals in acoustic delay lines.
- ❖ In an analog delay-line canceller the delay time and the pulse repetition period must be made equal. This is simplified in a digital MTI since the timing of the sampling of the bipolar video can be controlled readily by the timing of the transmitted pulse. Thus, different pulse repetition periods can be used without the necessity of switching delay lines of various lengths in and out. The echo signals for each inter- pulse period can be stored in the digital memory with reference to the time of transmission. This allows more elaborate stagger periods.
- ❖ The flexibility of the digital processor also permits more freedom in the selection and application of amplitude weightings for shaping the filters.
- ❖ It has also allowed the ready incorporation of the quadrature channel for elimination of blind phases.

In short, digital MTI has allowed the radar designer the freedom to take advantage of the full theoretical capabilities of doppler processing in practical radar systems.

## Limitations to MTI performance:

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the doppler signal processor. Instabilities of the transmitter and receiver, physical motions of the clutter, the finite time on target (or scanning modulation), and limiting in the receiver can all detract from the performance of an MTI radar. Before discussing these effects, let us see some relevant definitions

**MTI improvement factor**: The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

**Sub-Clutter visibility:** The ratio by which the target echo power may be weaker than the coincident clutter echo power and still be detected with specified detection and false alarm probabilities. All target radial velocities are assumed equally likely. A sub-clutter visibility of, for example, 30 dB implies that a moving target can be detected in the presence of clutter even though the clutter echo power is 1000 times the target echo power. Two radars with the same sub-clutter visibility might not have the same ability to detect targets in clutter if the resolution cell of one is greater than the other and accepts a greater clutter signal power; that is, both radars might reduce the clutter power equally, but one starts with greater clutter power because its resolution cell is greater and "sees" more clutter targets.

**Clutter visibility factor:** The signal-to-clutter ratio, after cancellation or doppler filtering, that provides stated probabilities of detection and false alarm.

**Clutter attenuation:** The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller. (The **clutter residue** is the clutter power remaining at the output of an MTI system.)

**Cancellation ratio:** The ratio of canceller voltage amplification for the fixed-target echoes received with a fixed antenna, to the.

**Inter clutter visibility:** This describes the ability of MTI radar to detect moving targets which occur in the relatively clear resolution cells between patches of strong clutter. Clutter echo power is not uniform, so if radar has sufficient resolution it can see targets in the clear areas between clutter patches. The higher the radar resolution, the better the inter clutter visibility.

## Pulse-Doppler Radar:

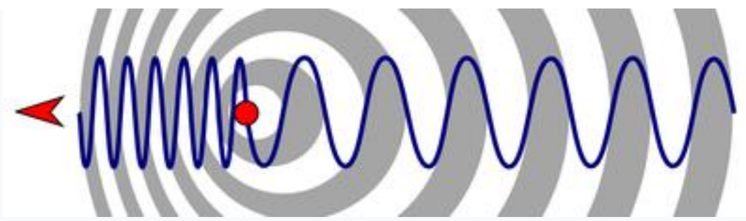
A **pulse-Doppler radar** is a radar system that determines the range to a target using pulse-timing techniques, and uses the Doppler effect of the returned signal to determine the target object's velocity. It combines the features of pulse radars and continuous-wave radars, which were formerly separate due to the complexity of the electronics.

Pulse-Doppler techniques also find widespread use in meteorological radars, allowing the radar to determine wind speed from the velocity of any precipitation in the air. Pulse-Doppler radar is also the basis of synthetic aperture radar used in radar astronomy, remote sensing and mapping. In air traffic control, they are used for discriminating aircraft from clutter.

### Range Measurement:

Pulse-Doppler systems measure the range to objects by measuring the elapsed time between sending a pulse of radio energy and receiving a reflection of the object. Radio waves travel at the speed of light, so the distance to the object is the elapsed time multiplied by the speed of light, divided by two - there and back.

### Velocity Measurement:



Change of wavelength caused by motion of the source

Pulse-Doppler radar is based on the Doppler effect, where movement in range produces frequency shift on the signal reflected from the target.

$$\text{Doppler frequency} = \frac{2 \times \text{transmit frequency} \times \text{radial velocity}}{C}.$$

Radial velocity is essential for pulse-Doppler radar operation. As the reflector moves between each transmit pulse, the returned signal has a phase difference, or phase shift, from pulse to pulse. This causes the reflector to produce Doppler modulation on the reflected signal.

Pulse-Doppler radars exploit this phenomenon to improve performance.

## Solved Problems:

1. The operating frequency of the radar is 3 G Hz. If the relative velocity of the target is 500 kmph, find out the Doppler shift .

$$f_d = 2\,\mathbf{V_r} / \lambda = 2v_r f / c = \frac{2 \times 500 \times 10^3 \times 3 \times 10^9}{3 \times 10^8 \times 3600} = 2.777 \text{ k Hz}$$

2. With a (CW) transmit frequency of 5 GHz, calculate the Doppler frequency seen by a stationary radar when the target radial velocity is 100km/h(62.5mph).

$$f_d = 2\,\mathbf{V_r} / \lambda = 2v_r f / c = \frac{2 \times 100 \times 10^3 \times 5 \times 10^9}{3 \times 10^8 \times 3600} = 0.925 \text{ k Hz}$$

3. The transmitter power is 1 KW and safe value of power which might be applied to a receiver is 10mW. Find the isolation between transmitter and receiver in dB. Suggest the appropriate isolator.

Isolation= 10 log [1 kw/10mW] = 10 log $[10^3/10^{-2}]$ = 10 log $10^5$ = >50 dB

4. Determine the angular frequency if the target is moving with a velocity of 2km/h and operating wavelength is 4cms.

Angular Frequency = $\omega_d = 4\pi \mathbf{V_r} / \lambda = [4 \pi \times 2 \times 10^3] / [4 \times 10^{-2} \times 3600] = 174$ rad/s

5. Calculate Doppler frequency shift ( f ) when the relative velocity of target with respect to radar is 50 knots at a transmitted frequency of 80MHZ.

1 knot = 1.852 km/h

50 knots= 92.6 km/h

$f_d = 2\,\mathbf{V_r} / \lambda = 2v_r f / c = 2 \times 92.6 \times 10^3 \times 80 \times 10^6 = 13.71$ Hz

$3 \times 10^8 \times 3600$

## Objective Type Questions:

1. In a CW radar
   - A. **Transmission & reception takes place simultaneously**
   - B. Reception takes place after transmission is over
   - C. Transmission is only for short duration
   - D. None of the above
2. The sign of doppler frequency shift depends upon
   - A. Phase of transmitted signal
   - B. Reference frequency
   - C. Direction of rotation of antenna
   - D. **Direction of relative velocity of target**
3. The doppler filter in a CW radar is primarily a
   - A. Low Pass Filter
   - B. High Pass Filter
   - C. **Band Pass Filter**
   - D. Band Elimination Filter
4. The doppler frequency shift is proportional to
   - A. Operating frequency
   - B. PRF
   - C. Relative velocity of target
   - D. **Both A & C above**
5. The doppler frequency shift caused by an aircraft circling at a speed of 100 m/sec around a radar operating at frequency of 3 G Hz is
   - A. 2 kHz
   - B. 20 kHz
   - **C. Zero**
   - D. 2 MHz
6. In a CW radar receiver the Local Oscillator is replaced by
   - A. Second detector
   - B. High accuracy mixer
   - C. Band pass filter
   - D. **Transmitted signal feedback**
7. The flicker noise in radar receiver increases with
   - A. Increase in Intermediate frequency
   - B. **Decrease in Intermediate frequency**
   - C. Increase in radar frequency
   - D. Both A & C above
8. A duplexer is used to

A. Couple two antennas to a transmitter without interference
B. Isolate the antenna from the local oscillator
C. Prevent interference between two antennas connected to a receiver
D. **Use single antenna for reception and transmission without interference**

9. Zero Intermediate frequency of CW radars causes
A. Thermal noise
B. Cancellation noise
C. **Flicker noise**
D. None of the above

10. In a CW-FM radar the type of modulation used is
A. Pulse amplitude modulation
B. Pulse width modulation
C. **Frequency modulation**
D. Pulse position modulation

11 In a CW-FM radar the range of a moving target depends upon
A. Doppler frequency shift
B. Difference in the frequency of transmitted & received frequency
C. Target Cross sectional area
D. **Both A & B above**

12. The significant limitation of single frequency CW-FM radar is
A. **Small unambiguous range**
B. Poor range resolution
C. Can't detect moving targets
D. Can detect one target at a time

13. If the upbeat & down beat frequencies of a CW radar are 100 Hz and 300 Hz respectively, the doppler frequency shift is
A. 400 Hz
**B. 100 Hz**
C. 200 Hz
D. 600 Hz

14. The maximum unambiguous range of a CW radar operating at two frequencies separated by 1 kHz is

| | | | |
|---|---|---|---|
| A. | 100 km | **B.** | **150 km** |
| C. | 200 km | D. | 300 km |

15. The upper frequency cutoff of Doppler filter of FM-CW radar is determined by the

| | | | |
|---|---|---|---|
| A. | Clutter strength | **B.** | **Maximum radial velocity expected of moving targets** |
| C. | Minimum radial velocity expected of moving targets | D. | Carrier frequency of the radar |

# Unit – 03

## Tracking Radar:

A tracking-radar system measures the coordinates of a target and provides data which may be used to determine the target path and to predict its future position. All or only part of the available radar data-range, elevation angle, azimuth angle, and Doppler frequency shift may be used in predicting future position; that is, a radar might track in range, in angle. In doppler, or with any combination. Almost any radar can be considered a tracking radar provided its output information is processed properly. But, in general, it is the method by which **angle tracking** is accomplished that distinguishes what is normally considered a **tracking radar** from any otherradar.

The Radar, which is used to track the path of one or more targets is known as **Tracking Radar**. In general, it performs the following functions before it starts the tracking activity.

- Target detection
- Range of the target
- Finding elevation and azimuth angles
- Finding Doppler frequency shift

So, Tracking Radar tracks the target by tracking one of the three parameters — range, angle, Doppler frequency shift. Most of the Tracking Radars use the **principle of tracking in angle**. Now, let us discuss what angular tracking is.

## Angular Tracking:

The pencil beams of Radar Antenna perform tracking in angle. The axis of Radar Antenna is considered as the reference direction. If the direction of the target and reference direction is not same, then there will be **angular error**, which is nothing but the difference between the two directions.

If the angular error signal is applied to a servo control system, then it will move the axis of the Radar Antenna towards the direction of target. Both the axis of Radar Antenna and the direction of target will **coincide** when the angular error is zero. There exists a feedback mechanism in the Tracking Radar, which works until the angular error becomes zero.

Following are the **two techniques**, which are used in angular tracking.

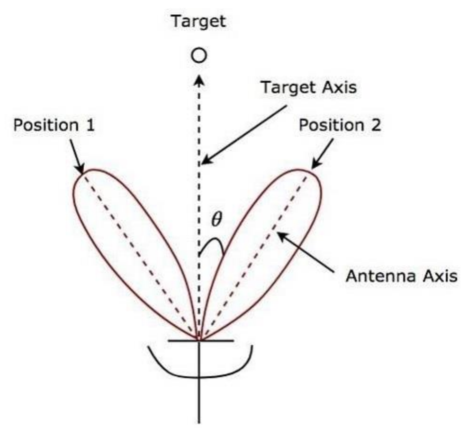
- Sequential Lobing
- Conical Scanning

Now, let us discuss about these two techniques one by one.

## Sequential Lobing:

If the Antenna beams are switched between two patterns alternately for tracking the target, then it is called **sequential lobing**. It is also called sequential switching and lobe switching. This technique is used to find the angular error in one coordinate. It gives the details of both magnitude and direction of angular error.

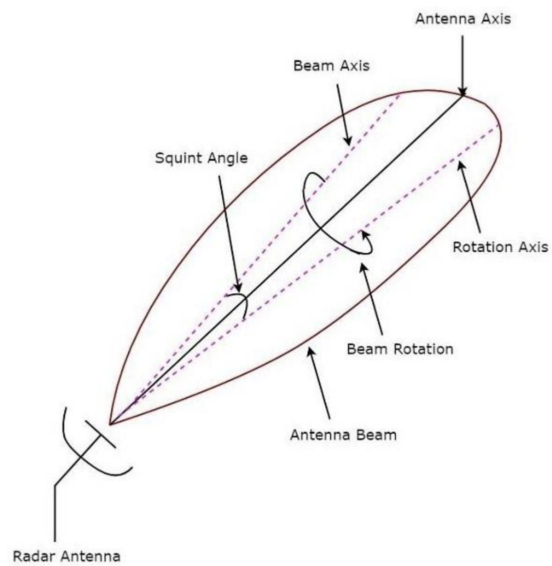
Following figure shows an example of sequential lobing in **polar coordinates**.




As shown in the figure, Antenna beams switch between Position 1 and Position 2 alternately. Angular error θ is indicated in the above figure. Sequential lobing gives the position of the target with high accuracy. This is the main **advantage** of sequential lobing.

## Conical Scanning:

If the Antenna beam continuously rotates for tracking a target, then it is called conical scanning. Conical scan modulation is used to find the position of the target. Following figure shows an example of conical scanning.




**Squint angle** is the angle between beam axis and rotation axis and it is shown in the above figure. The echo signal obtained from the target gets modulated at a frequency equal to the frequency at which the Antenna beam rotates.

The angle between the direction of the target and the rotation axis determines the **amplitude of the modulated signal**. So, the conical scan modulation has to be extracted from the echo signal and then it is to be applied to servo control system, which moves the Antenna beam axis towards the direction of the target.

## Monopulse Tracking Radar:

The conical-scan and sequential-lobing tracking radars require a minimum number of pulses in order to extract the angle-error signal. In the time interval during which a measurement is made with either sequential lobing or conical scan, the train of echo pulses must contain no amplitude-modulation components other than the modulation produced by scanning. If the echo pulse-train did contain additional modulation components, caused, for example, by a fluctuating target cross section, the tracking accuracy might he degraded, especially if the frequency components of the fluctuations were at or near the conical-scan frequency or the sequential-lobing rate. The effect of the fluctuating echo can be sufficiently serious in some applications to severely limit the accuracy of those tracking radars which require many pulses to be processed in extracting the error signal.

Pulse-to-pulse amplitude fluctuations of the echo signal have no effect on tracking accuracy if the angular measurement is made **on the basis of one pulse** rather than many. There are several methods by which angle-error information might be obtained with only a single pulse. More than one antenna beam is used simultaneously in these methods, in contrast to the conical-scan or lobe-switching tracker, which utilizes one antenna beam on a time- shared basis. The angle of arrival of the echo signal may be determined in a single-pulse system by measuring the relative phase or the relative amplitude of the echo pulse received in each beam. The names simultaneous lobing and monopulse are used to describe those tracking techniques which derive angle-error information on the basis of a single pulse. The widely used monopulse techniques are

1. **Amplitude-comparison monopulse.**
2. **Phase-comparison monopulse.**

## Amplitude-comparison Monopulse:

The amplitude-comparison monopulse employs two overlapping antenna patterns to obtain the angular error in one coordinate. The two overlapping antenna beams may be generated with a single reflector or with a lens antenna illuminated by two adjacent feeds. (A cluster of four feeds may be used if both elevation- and azimuth-error signals are wanted.) The sum of the two antenna patterns of Fig. (a) is shown in Fig. (b), and the difference in Fig. (c).

The sum patterns is used for transmission, while both the sum pattern and the difference pattern are used on reception. The signal received with the difference pattern provides the magnitude of the angle error. The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal. Signals received from the sum and the difference patterns are amplified separately and combined in a phase-sensitive detector to produce the error-signal characteristic shown in Fig.(d).

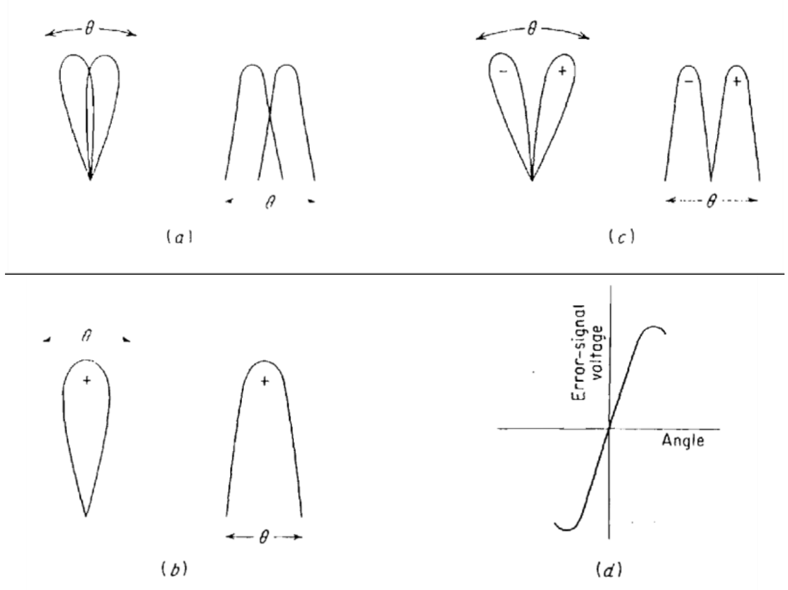



**Figure: Monopulse antenna patterns and error signal. Left-hand diagrams in (a-c) are in polar coordinates; right-hand diagrams are in rectangular coordinates. (a) Overlapping antenna patterns; (b) sum pattern; (c) difference pattern; (d) product (error) signal.**

**A** block diagram of the amplitude-comparison-monopulse tracking radar for a single angular coordinate is shown in Fig. 6.10. The two adjacent antenna feeds are connected to the two arms of a hybrid junction such as a "magic-T," a " rat race," or a short-slot coupler. The sum and difference signals appear at the two other arms of the hybrid. On reception, the outputs of the sum arm and the difference arm are each heterodyned to an intermediate frequency and amplified as, in any super-heterodyne receiver. The transmitter is connected to the sum arm. Range information is also extracted from the sum channel. A duplexer is included in the sum arm for the protection of the receiver. The output of the phase-sensitive detector is an error signal whose magnitude is proportional to the angular error and whose sign is proportional to the direction.

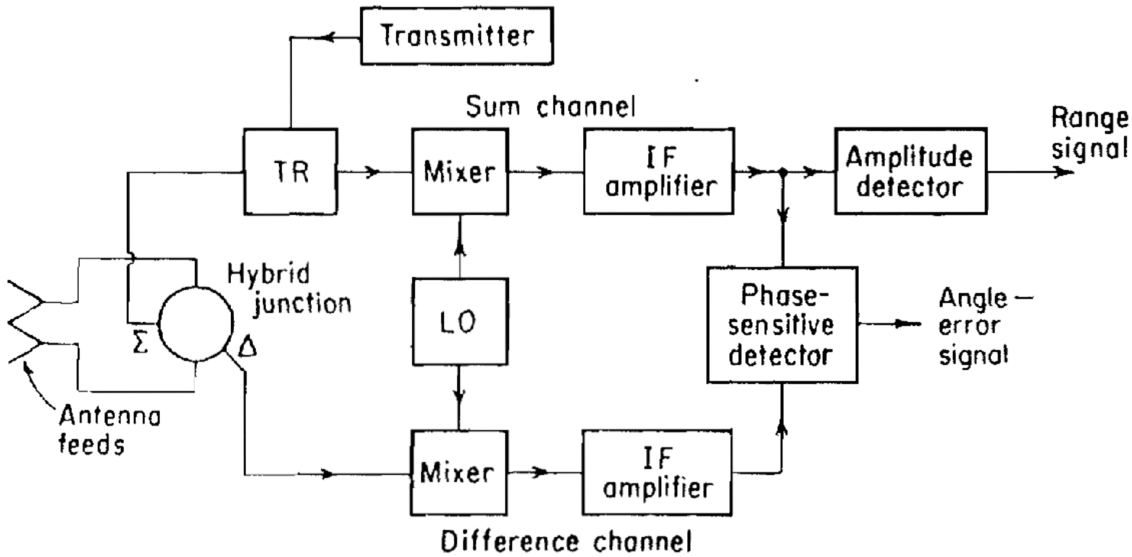



**Figure: Block diagram of amplitude-comparison monopulse radar (one angular coordinate).**

The output of the monopulse radar is used to perform automatic tracking. The angular error signal actuates a servo-control system to position the antenna, and the range output from the sum channel feeds into an automatic-range-tracking unit. The sign of the difference signal (and the direction of the angular error) is determined by comparing the phase of the difference signal with the phase of the sum signal. If the sum signal in the IF portion of the receiver were $A_s \cos \omega_{IF} t$, the difference signal would be either $A_d \cos \omega_{IF} t$ or $-A_d \cos \omega_{IF} t$ ($A_s > 0, A_d > 0$), depending on which side of center is the target. Since $-A_d \cos \omega_{IF} t = A_d \cos \omega_{IF} (t + \pi)$, the sign of the difference signal may be measured by determining whether the difference signal is in phase with the sum or $180^0$ out of phase.

## Phase Comparison Monopulse:

The tracking techniques discussed thus far in this chapter were based on a comparison of the amplitudes of echo signals received from two or more antenna positions. .The sequential-lobing and conical-scan techniques used a single, time-shared antenna beam, while the mono pulse technique used two or more simultaneous beams The difference in amplitudes in the several antenna positions was proportional to the angular error. **The angle of arrival (in one coordinate) may**

**also be determined by comparing the phase difference between the signals from two separate antennas.** Unlike the antennas of amplitude- comparison trackers. those used in phase-comparison systems are not offset from the axis. The individual boresight axes of the antennas are parallel, causing the (far- field) radiation to illuminate the same volume in space. The amplitudes of the target echo signals are essentially the same from each antenna beam, but the phases are different. A tracking radar which operates with phase information is similar to an active interferometer and might be called an interferometer radar. It has also been called simultaneous phase comparison radar, or phase-comparison monopulse. The latter term is the one which will be used here.

In Fig. below two antennas are shown separated by a distance d. The distance to the target is R and is assumed large compared with the antenna separation d. The line of sight to the

target makes an angle θ to the perpendicular bisector of the line joining the two antennas. The distance from antenna 1 to the target is target makes an angle θ to the perpendicular bisector of the line joining the two antennas.

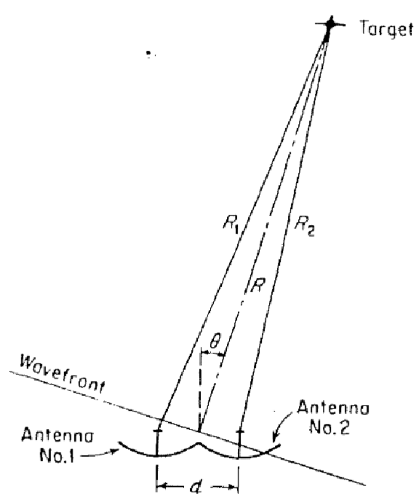



**Figure: Wavefront phase relationships in phase-comparison monopulse radar.**

The distance from antenna 1 to the target is

$$R_1 = R + \frac{d}{2}\sin\theta$$

and the distance from antenna 2 to the target is

$$R_2 = R - \frac{d}{2}\sin\theta$$

The phase difference between the echo signals in the two antennas is approximately

$$\Delta\phi = \frac{2\pi d}{\lambda}\sin\theta$$

For small angles where sin θ = θ, the phase difference is a linear function of the angular error and may be used to position the antenna via a servo-control loop.

Although tracking radars based upon the phase-comparison monopulse principle have been built and operated, this technique has not been as widely used as some of the other angle-tracking methods. The sum signal has higher sidelobes because the separation between the phase centers of the separate antennas is large. (These high sidelobes are the result of grating lobes similar to those produced in phased arrays.) The problem of high sidelobes can be reduced by overlapping the antenna apertures. With reflector antennas, this results in a loss of angular sensitivity and antenna gain.

## Tracking in Range:

In most tracking-radar applications the target is continuously tracked in range as well as in angle. The technique for automatically tracking in range is based on the split range gate. Two range gates are generated as shown in Fig.. One is the **early gate**, and the other is the **late gate**. The echo pulse is shown in Fig.-a, the relative position of the gates at a particular instant in Fig. - b, and the error signal in Fig. - c. The portion of the signal energy contained in the early gate is less than that in the late gate. If the outputs of the two gates are subtracted, an error signal (Fig. - c) will result which may be used to reposition the center of the gates. The magnitude of the error signal is a measure of the difference between the center of the pulse and the center of the gates. The sign of the error signal determines the direction in which the gates must be repositioned by a feedback-control system. When the error signal is zero. the range gates are centered on the pulse.

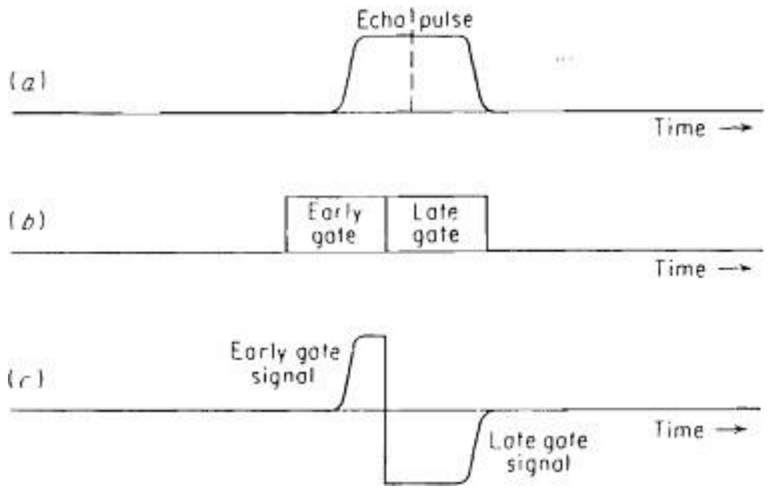



**Figure: Split-range-gate tracking. (I) Echo pulse; (h) early-late range gates; (c) difference signal between early and late range gates.**

The range gating necessary to perform automatic tracking offers several advantages as by-products. It isolates one target, excluding targets at other ranges. This permits the boxcar generator to be employed. Also, range gating improves the signal-to-noise ratio since it eliminates the noise from the other range intervals. Hence the width of the gate should be sufficiently narrow to minimize extraneous noise. On the other hand, it must not be so narrow that an appreciable fraction of the signal energy is excluded. A reasonable compromise is to make the gate width of the order of the pulse width.

A target of finite length can cause noise in range-tracking circuits in an analogous manner to angle-fluctuation noise (glint) in the angle-tracking circuits. Range-tracking noise depends on the length of the target and its shape

## Angular Resolution:

Radar angular resolution is the minimum distance between two equally large targets at the same range which radar is able to distinguish and separate to each other.

### Angular Resolution as Antenna Parameter:

The angular resolution characteristics of radar are determined by the antenna beamwidth represented by the −3 dB angle Θ which is defined by the half-power (−3 dB) points. The half-power points of the antenna radiation pattern (i.e. the −3 dB beamwidth) are normally specified as the limits of the antenna beamwidth for the purpose of defining angular resolution; two identical targets at the same distance are, therefore, resolved in angle if they are separated by more than the antenna −3 dB beamwidth.

An important remark has to be made immediately: the smaller the beamwidth Θ, the higher the directivity of the radar antenna. The angular resolution as a distance between two targets calculate the following formula:

$$S_A \geq 2R \cdot \sin\frac{\Theta}{2}$$

Where;

Θ = antenna beamwidth (Theta)

SA = angular resolution as a distance between two targets

R = slant range aim - antenna [m]

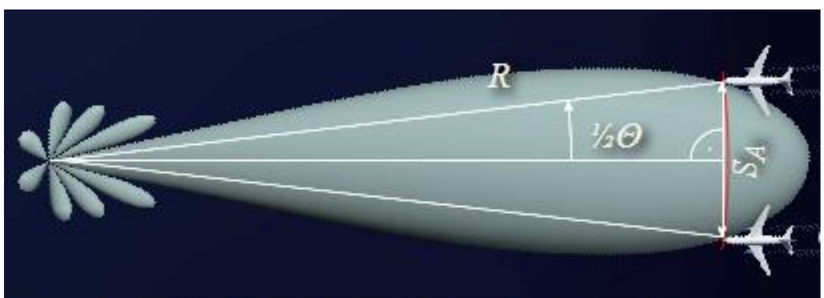



*Figure 1: The distance $S_A$ depends on the slant-range*

The angular resolution of targets on an analog PPI-scope, in practical terms, is dependent on the operator being able to distinguish the two targets involved. Systems having Target-Recognition features can improve their angular resolution. Cause such systems are able to compare individual Target-Pulse-Amplitudes.

In 3D radars, a resolution in the elevation angle can also be measured. Here, the same method is applicable as in the azimuth resolution. As angle Θ is used the vertical beamwidth then.

## Angular Resolution using Lidar:

In certain cases (for example, Lidar), it is easier to determine the angular resolution don't use the half-power beamwidth but optical rules. The resolution of an optical system is defined by the angular separation between two similar point-like objects, the main maximum of the image of one point-like object being within the first minimum of the image of the other object.

This definition based on radar means: The angular resolution of radar is defined as the angular distance between the first minimum of the antenna pattern (next to the main lobe) and the maximum of the main lobe (null angle or the half beamwidth between nulls)

To calculate the beamwidth you can use the relationship:

$$\Theta = K \frac{\lambda}{D}$$

Where;

$\lambda$ = free-space wavelength

D = aperture dimension

K = beamwith factor

Assuming a linear phase distribution, each amplitude distribution has a corresponding beamwidth factor, expressed either in radians or in Radiant. The beamwidth factor depends on the antenna type and varies from 0.89 rad (56 degrees) for an ideal reflector antenna, up to 2 rad (114 degrees). For the null angle of a synthetic aperture the beamwidth factor is 1.220 rad (70 degrees).

The null angle thus only refers to half the width of the main lobe. The half-power beamwidth and the beamwidth between nulls relate to the entire width of the main lobe (see Figure 2). Both angles (zero angle and half-power beamwidth) are thus similar in size but not equal. However, the difference in size can be neglected in practice approximately. However, when applying the formula (1), it is necessary to consider the relation of whether the angle used is only half the width of the main lobe or both halves. This confounds since the variable name «theta» is commonly used for all these angles in the literature.

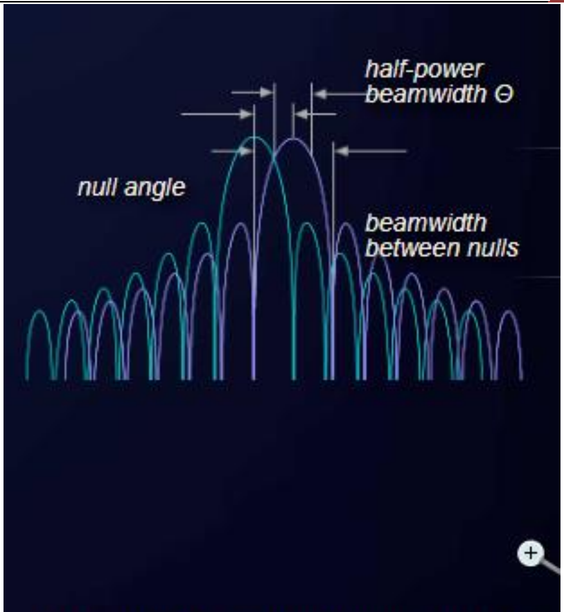


Figure 2: half-power beamwidth vs. null angle (half beamwidth between nulls)

## Cross Range Resolution:

Due to the computational postprocessing, the synthetic aperture radars (SAR's) resolution capability has completely different contexts than that of a classical radar with a real antenna. Specifying an angle for the beamwidth is impractical because it cannot be measured but only calculated. In the SAR, a distance is measured perpendicular to the direction of the movement of the radar platform. The resolution in the direction of motion is thus orthogonal to the radar beam and the range measuring and is, therefore, called cross-range resolution. In contrast to the real aperture, the cross range resolution improves with the increase in the half-power beamwidth of the real antenna. The radar will only process echo signals from the object that is detected by the antenna during a flyby in all measurements. There are the more measurement results, the larger the half-power beamwidth of the real antenna. The more single echo signals flow into the processing, the better will be the angular resolution. Since the so-called “footprint” of the radar increases with the distance, the synthetic aperture increases too, and the angular resolution at a larger distance is better than at the close range. This compensates for the deterioration of resolution due to the larger distance. In contrast to the real aperture, the cross-range resolution of the SAR is therefore approximately constant with increasing range.

## Angular Resolution using CW Radar:

At short distances using a CW radar extreme accuracies are possible with very little effort. Therefore, such radars are often used for speeds gauges on the road. The angular resolution is determined according to equation (1) by the half-power beamwidth Θ of the antenna and the distance R to the object to be measured.

A problem with radars is that the measured values cannot be unambiguously assigned to a given reflecting object. Depending on the kind of modulation of these FMCW radars, separation of two simultaneously reflecting objects is often not possible. If the distance to the measurement object is large, then the antenna pattern widens. If then two cars are located in the main lobe, a separation is not possible. In this case, the measurement is invalid.

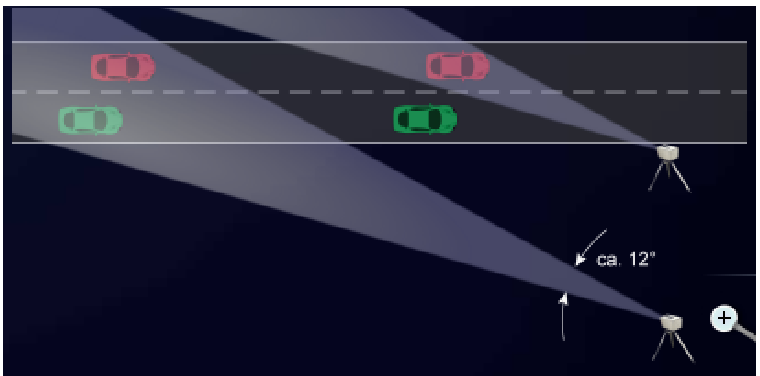



Figure 3: Influence of the distance on the angle resolution.

## Matched Filter:

In this unit some aspects of the problem of detecting radar signals in the presence of noise will be considered. Noise ultimately limits the capability of any radar. The detection of signals in the presence of clutter is always a challenging task for the design engineers.

**A network whose frequency-response function maximizes the output peak-signal-to- mean noise (power) ratio is called a matched filter. This criterion, or its equivalent, is used for the design of almost all radar receivers.**

The frequency-response function, denoted **H(f)**, expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.

The magnitude I **H(**f)I of the frequency-response function is the receiver amplitude pass band characteristic.

**Case I:** If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio.

**Case-II:** On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy. The net result is again a lowered signal-to-noise ratio.

Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer. The rule of thumb quoted in pulse radar practice is that the receiver bandwidth **B** should be approximately equal to the reciprocal of the pulse width **T.**

As we shall see later, this is a reasonable approximation for pulse radars with conventional Super heterodyne receivers. It is not generally valid for other waveforms, however, and is mentioned to illustrate in a qualitative manner the effect of the receiver characteristic on signal-to-noise ratio. The exact specification of the optimum receiver characteristic involves the frequency-response function and the shape of the received waveform.

- ❖ The receiver frequency-response function, is assumed to apply from the antenna terminals to the output of the IF amplifier.
- ❖ The second detector and video portion of the well-designed radar super heterodyne receiver will have negligible effect on the output signal-to-noise ratio if the receiver is designed as a matched filter.)
- ❖ Narrow banding is most conveniently accomplished in the IF. The bandwidths of the RF and mixer stages of the normal super heterodyne receiver are usually large compared with the IF bandwidth.

Therefore the **frequency-response function** of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.

The IF amplifier may be considered as a filter with gain. The response of this filter as a function of frequency is the property of interest.

**Frequency-response function:**

For a received waveform S(t) with a given ratio of signal energy E to noise energy **No** (or noise power per hertz of bandwidth), the frequency-response function of the linear, time- invariant filter which maximizes the output peak-signal-to-mean-noise (power) ratio is given by

$$\mathbf{H(f) = G_a S^*(f) \exp(-j2\pi f t_1)} \text{.....................................} (7.1)$$

where $S(f) = \int S(t) \exp(-j2\pi f t_1)\, \mathbf{dt}$ = voltage spectrum (Fourier transform) of input signal $S^*(f)$ = complex conjugate of S(f)= S(-f)

$t_1$ = fixed value of time at which signal is observed to be maximum

$\mathbf{G_a}$= constant equal to maximum filter gain (generally taken to be unity)

The noise that accompanies the signal is assumed to be stationary and to have a uniform Spectrum (white noise). It need not be Gaussian.

If the noise is not white, Eq. (7.1) may be modified and will be discussed later in this section. The filter whose frequency-response function is given by Eqn (7.1) has been called the North filter, the conjugate filter, or more usually the matched filter.

The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift $\exp(-j2\pi f t_1)$. This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay. A time delay is necessary in the specification of the filter for reasons of physical realizability since there can be no output from the filter until the signal is applied.

The frequency spectrum of the received signal may be written as an amplitude spectrum $|S(f)|$ and a phase spectrum $\exp[-j\Phi_s(f)]$. The matched filter frequency-response function may similarly be written in terms of its amplitude and phase spectra $|H(f)|$ and $\exp[-j\Phi_m(f)]$.

$$S(f) = |S(f)| \exp[-j\Phi_s(f)] \qquad (7.2)$$

$$S^*(f) = |S(f)| \exp[+j\Phi_s(f)] \qquad (7.3)$$

$$H(f) = |H(f)| \exp[-j\Phi_m(f)] \qquad (7.4)$$

Ignoring the constant $\mathbf{G_a}$, substituting Eqn (7.3) and Eqn(7.4) in Eqn (7.1) for the matched filter may then be written as

$|H(f)| \exp[-j\Phi_m(f)] = |S(f)| \exp[j\Phi_s(f) - j2\pi f t_1]$ (7.5)

Or $|H(f)| = |S(f)|$ (7.6)

and $\Phi_m(f) = -\Phi_s(f) + 2\pi f t_1$ (7.7)

**Thus the amplitude spectrum of the matched filter is same as the amplitude spectrum of input signal.**

The phase spectrum of the matched filter is the negative of phase spectrum of input signal plus a phase shift proportional to the frequency.

Let h(t) be the **impulse response of the matched filter.** Therefore h(t) is the inverse Fourier transform of H(f). by definition

$h(t) = \int H(f) \exp[j2\pi f t]\, df$ (7.8)

Substituting for H(f) from eqn (7.1)

$h(t) = G_a \int S^*(f) \exp\{-j2\pi f(t_1 - t)\}\, df$ (7.9)

since $S^*(f) = S(-f)$

$h(t) = G_a \int S^*(f) \exp\{j2\pi f (t_1 - t]\}\, df$ (7.10)

from the definition of Fourier transform **h(t)** can also be written

$$\mathbf{h(t) = G_a s(t_1 - t)} \qquad (7.11)$$

It can be seen that the impulse response of matched filter is the image of the received signal run backwards in time starting from fixed time ‘t1’

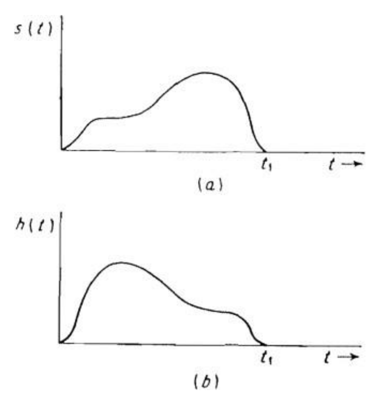



**Figure:** (a) Received waveform s ( t ) ;

(b) impulse response h(t) of the matched filter.

## Derivation of the matched-filter characteristic:

The frequency-response function of the matched filter has been derived by a number of authors using either the calculi of variations or the Schwartz inequality. Here we shall derive the matched-filter frequency response function using the Schwartz inequality.

We assume that the input noise is white noise.(Uniform Spectral density)

By definition matched filter is used to maximse

$R_f = |s_o(t)|^2_{max} / N$ (7.12)

Where $|s_o(t)|_{max}$ is the maximum value of output signal voltage. N is the mean power noise at the output.

Let H(f) be the frequency response function of the matched filter. By definition the output signal of matched filter can be written as

$s_o(t) = |\int S(f)\, H(f) \exp[j2\pi f t_1]\, df|$ (7.13)

$N = N_o/2 \int |H(f)^2|\, df$ (7.14)

Where $N_o$ is the input noise power per unit bandwidth. The factor ½ appears before the integral because of limits extended from $-\infty$ to $+\infty$ where No is defined for positive values only.

Substituting eqns (17.3) and (7.14) in eqn (7.12)

$R_f = [|\int S(f)\, H(f) \exp[j2\pi f t_1]\, df|]^2 \;/\; [N_o/2 \int |H(f)^2|\, df]$ (7.15)

Schwartz's inequality states that if P and Q are two complex functions, then

$\int P^*P\, dx \,.\, \int Q^*Q\, dx \geq |P^*Q\, dx|^2$ (7.16)

The equality sign applies when P = kQ, where k is a constant

Let $P = S(f) \exp[j2\pi f t_1]$ (7.17)

and $Q = H(f)$ (7.18)

Then $\int P^*P\, dx = \int |P|^2\, dx$

$\int |P^*Q\, dx|^2 = |S(f)\, H(f) \exp[j2\pi f t_1]\, df|^2$ (7.19)

$\int P^*P\, dx = \int |S(f)|^2\, df$ (omitting the phase) (7.20)

$\int Q^*Q\, dx = \int |H(f)|^2\, df$ (7.21)

$\int |H(f)|^2\, df \,.\, \int |S(f)|^2\, df \leq |S(f)\, H(f) \exp[j2\pi f t_1]\, df|^2$ (7.22)

Dividing both sides of above equation by $N_o/2 \int |H(f)|^2 \, df$

$$\frac{\int |S(f)|^2 \, df}{N_o/2} \leq R_f \qquad (7.23)$$

From Parseral's Theorem we have

Signal Energy $E = \int |S(f)|^2 \, df = \int |s(t)|^2 \, dt$

Eqn 7.23 can be written as

$$\frac{2E}{N_o} \leq R_f \qquad (7.24)$$

The equality sign is applicable if P= kQ

$$R_f = \frac{2E}{N_o} \qquad (7.25)$$

$$H(f) = k\, S(f) \exp(\, j2\pi f t_1) \qquad (7.26)$$

Where k is constant and assumed to be the gain $G_a$ Since

$S^*(f) = S(-f)$, eqn 26 becomes

$$\mathbf{H(f) = G_a S^*(f) \exp(-j2\pi f t_1)} \qquad (7.27)$$

## The Radar Ambiguity Function:

The radar ambiguity function is defined as the absolute value of the envelope of the output of a matched filter when the input to the filter is a Doppler-shifted version of the original signal, to which the filter was matched. Ambiguity functions are usually analyzed on a single pulse basis.

The radar ambiguity function represents the output of the matched filter, and it describes the interference caused by range and/or Doppler of a target when compared to a reference target of equal RCS. The ambiguity function evaluated at is equal to the matched filter output that is matched perfectly to the signal reflected from the target of interest. In other words, returns from the nominal target are located at the origin of the ambiguity function. Thus, the ambiguity function at nonzero and represents returns from some range and Doppler different from those for the nominal target. The radar ambiguity function is normally used by radar designers as a means of studying different waveforms. It can provide insight about how different radar waveforms may be suitable for the various radar applications. It is also used to determine the range and Doppler resolutions for a specific radar waveform. The three-dimensional (3-D) plot of the ambiguity function versus frequency and time delay is called the radar ambiguity diagram. The radar ambiguity function for the signal is defined as the modulus squared of its 2-D correlation function, i.e., . More precisely,

$$|\chi(\tau;f_d)|^2 = \left|\int_{-\infty}^{\infty} s(t)s^*(t+\tau)e^{j2\pi f_d t}dt\right|^2$$

In this notation, the target of interest is located at , and the ambiguity diagram is centered at the same point. Note that some authors define the ambiguity function as . In this book, is called the uncertainty function. Denote as the energy of the signal ,

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

**We will now list the properties for the radar ambiguity function:**

1) The maximum value for the ambiguity function occurs at (τ fd) = (0,0 )and is equal to ,

$$max\{|\chi(\tau;f_d)|^2\} = |\chi(0;0)|^2 = (2E)^2$$

$$|\chi(\tau;f_d)|^2 \le |\chi(0;0)|^2$$

2) The ambiguity function is symmetric,

$$|\chi(\tau;f_d)|^2 = |\chi(-\tau;-f_d)|^2$$

3) The total volume under the ambiguity function is constant,

$$\int\int|\chi(\tau;f_d)|^2 \ d\tau \ df_d = (2E)^2$$

4) If the function is the Fourier transform of the signal , then by using Parseval's theorem we get

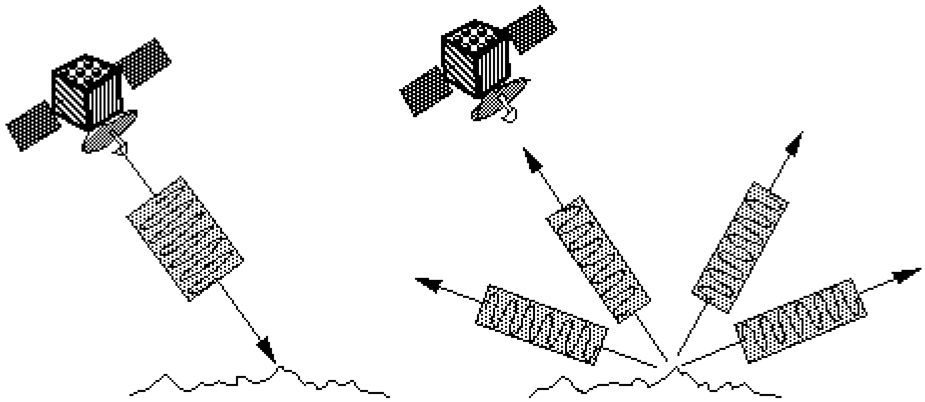
$$|\chi(\tau;f_d)|^2 = \left|\int S^*(f)S(f-f_d)e^{-j2\pi f\tau}df\right|^2$$

# Unit – 04

## Imaging Radar:

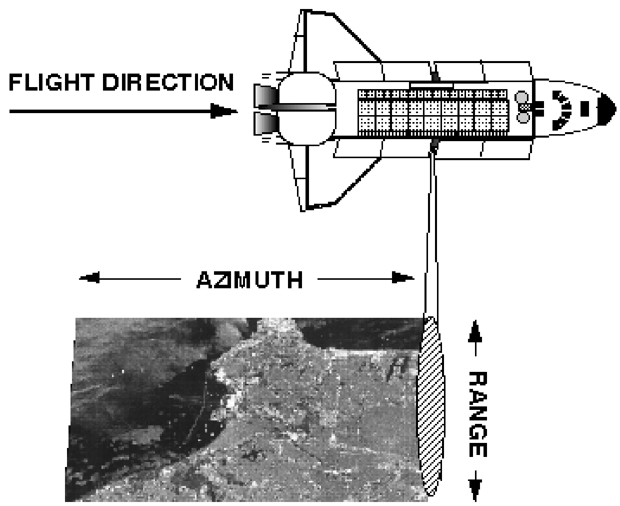
An imaging radar works very like a flash camera in that it provides its own light to illuminate an area on the ground and take a snapshot picture, but at radio wavelengths. A flash camera sends out a pulse of light (the flash) and records on film the light that is reflected back at it through the camera lens. Instead of a camera lens and film, a radar uses an antenna and digital computer tapes to record its images. In a radar image, one can see only the light that was reflected back towards the radar antenna.

A typical radar (RAdio Detection and Ranging) measures the strength and round-trip time of the microwave signals that are emitted by a radar antenna and reflected off a distant surface or object. The radar antenna alternately transmits and receives pulses at particular microwave wavelengths (in the range 1 cm to 1 m, which corresponds to a frequency range of about 300 MHz to 30 GHz) and polarizations (waves polarized in a single vertical or horizontal plane). For an imaging radar system, about 1500 high-power pulses per second are transmitted toward the target or imaging area, with each pulse having a pulse duration *(pulse width)* of typically 10-50 microseconds (us). The pulse normally covers a small band of frequencies, centered on the frequency selected for the radar. Typical bandwidths for an imaging radar are in the range 10 to 200 MHz. At the Earth's surface, the energy in the radar pulse is scattered in all directions, with some reflected back toward the antenna. This backscatter returns to the radar as a weaker radar echo and is received by the antenna in a specific polarization (horizontal or vertical, not necessarily the same as the transmitted pulse). These echoes are converted to digital data and passed to a data recorder for later processing and display as an image. Given that the radar pulse travels at the speed of light, it is relatively straightforward to use the measured time for the roundtrip of a particular pulse to calculate the distance or range to the reflecting object. The chosen pulse bandwidth determines the resolution in the range (cross-track) direction. Higher bandwidth means finer resolution in this dimension.



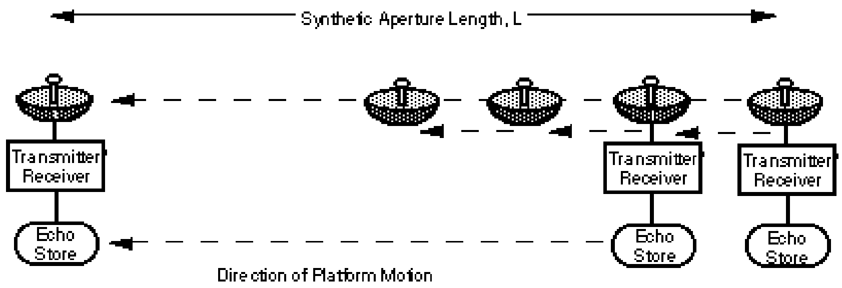
*Radar transmits a pulse Measures reflected echo (backscatter )*

In the case of imaging radar, the radar moves along a flight path and the area illuminated by the radar, or *footprint*, is moved along the surface in a swath, building the image as it does so.




*Building up a radar image using the motion of the platform*

The length of the radar antenna determines the resolution in the azimuth (along-track) direction of the image: the longer the antenna, the finer the resolution in this dimension. *Synthetic Aperture Radar (SAR)* refers to a technique used to synthesize a very long antenna by combining signals (echoes) received by the radar as it moves along its flight track. Aperture means the opening used to collect the reflected energy that is used to form an image. In the case of a camera, this would be the shutter opening; for radar it is the antenna. A *synthetic* aperture is constructed by moving a real aperture or antenna through a series of positions along the flight track.




*Constructing a Synthetic Aperture*

As the radar moves, a pulse is transmitted at each position; the return echoes pass through the receiver and are recorded in an 'echo store.' Because the radar is moving relative to the ground, the returned echoes are Doppler-shifted (negatively as the radar approaches a target; positively as it moves away). Comparing the Doppler-shifted frequencies to a reference frequency allows many returned signals to be "focused" on a single point, effectively increasing the length of the antenna that is imaging that particular point. This focusing operation, commonly known as SAR processing, is now done digitally on fast computer systems. The trick in SAR processing is to correctly match the variation in Doppler frequency for each point in the image: this requires very precise knowledge of the relative motion between the platform and the imaged objects (which is the cause of the Doppler variation in the first place).

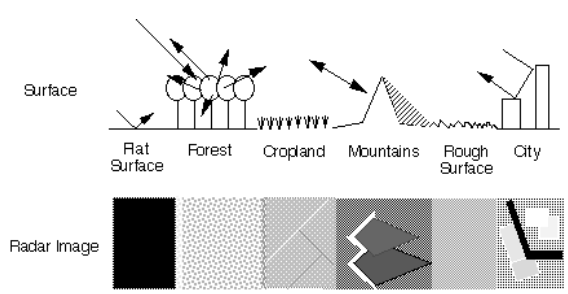
Synthetic aperture radar is now a mature technique used to generate radar images in which fine detail can be resolved. SARs provide unique capabilities as an imaging tool. Because they provide their own illumination (the radar pulses), they can image at any time of day or night, regardless of sun illumination. And because the radar wavelengths are much longer than those of visible or infrared light, SARs can also "see" through cloudy and dusty conditions that visible and infrared instruments cannot.

## What is a Radar Image?

Radar images are composed of many dots, or picture elements. Each pixel (picture element) in the radar image represents the radar backscatter for that area on the ground: darker areas in the image represent low backscatter, brighter areas represent high backscatter. Bright features mean that a large fraction of the radar energy was reflected back to the radar, while dark features imply that very little energy was reflected. Backscatter for a target area at a particular wavelength will vary for a variety of conditions: size of the scatterers in the target area, moisture content of the target area, polarization of the pulses, and observation angles. Backscatter will also differ when different wavelengths are used.

Scientists measure backscatter, also known as radar cross section, in units of area (such as square meters). The backscatter is often related to the size of an object, with objects approximately the size of the wavelength (or larger) appearing bright (i.e. rough) and objects smaller than the wavelength appearing dark (i.e. smooth). Radar scientists typically use a measure of backscatter called normalized radar cross section, which is independent of the image resolution or pixel size. Normalized radar cross section (sigma0.) is measured in decibels (dB). Typical values of sigma0. for natural surfaces range from +5dB (very bright) to -40dB (very dark).

A useful rule-of-thumb in analyzing radar images is that the higher or brighter the backscatter on the image, the rougher the surface being imaged. Flat surfaces that reflect little or no microwave energy back towards the radar will always appear dark in radar images. Vegetation is usually moderately rough on the scale of most radar wavelengths and appears as grey or light grey in a radar image. Surfaces inclined towards the radar will have a stronger backscatter than surfaces which slope away from the radar and will tend to appear brighter in a radar image. Some areas not illuminated by the radar, like the back slope of mountains, are in shadow, and will appear dark. When city streets or buildings are lined up in such a way that the incoming radar pulses are able to bounce off the streets and then bounce again off the buildings (called a double- bounce) and directly back towards the radar they appear very bright (white) in radar images. Roads and freeways are flat surfaces so appear dark. Buildings which do not line up so that the radar pulses are reflected straight back will appear light grey, like very rough surfaces.

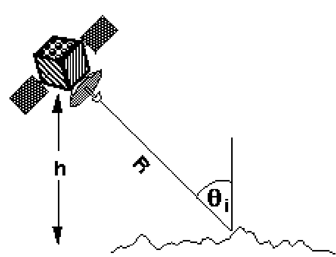



*Imaging different types of surface with radar*

Backscatter is also sensitive to the target's electrical properties, including water content. Wetter objects will appear bright, and drier targets will appear dark. The exception to this is a smooth body of water, which will act as a flat surface and reflect incoming pulses away from a target; these bodies will appear dark.

Backscatter will also vary depending on the use of different polarization. Some SARs can transmit pulses in either horizontal (H) or vertical (V) polarization and receive in either H or V, with the resultant combinations of HH (Horizontal transmit, Horizontal receive), VV, HV, or VH. Additionally, some SARs can measure the phase of the incoming pulse (one wavelength = 2pi in phase) and therefore measure the phase difference (in degrees) in the return of the HH and VV signals. This difference can be thought of as a difference in the roundtrip times of HH and VV signals and is frequently the result of structural characteristics of the scatterers. These SARs can also measure the correlation coefficient for the HH and VV returns, which can be considered as a measure of how alike (between 0/not alike and 1/alike) the HH and VV scatterers are.

Different observations angles also affect backscatter. Track angle will affect backscatter from very linear features: urban areas, fences, rows of crops, ocean waves, fault lines. The angle of the radar wave at the Earth's surface (called the incidence angle) will also cause a variation in the backscatter: low incidence angles (perpendicular to the surface) will result in high backscatter; backscatter will decrease with increasing incidence angles.




*Radar backscatter is a function of incidence angle, (theta)i*

## Resolution Concept:

Resolution is a measure used to describe the sharpness and clarity of an image or picture. It is often used as a metric for judging the quality of monitors, printers, digital images and various other hardware and software technologies.

## Pulse Compression:

Pulse compression allows a radar system to transmit a pulse of relatively long duration and low peak power to attain the range resolution and detection performance of a short-pulse, high-peak power system. This is accomplished by coding the RF carrier to increase the bandwidth of the transmitted waveform and then compressing the received echo waveform.

Pulse compression is a method for improving the range resolution of pulse radar. This method is also known as intra-pulse modulation (modulation on pulse, MOP) because the transmitted pulse got a time-dependent modulation internally. The term CHIRP–Radar is often used in publications (Compressed, HIgh-Resolution Pulse, CHIRP). Pulse compression combines the energetic advantages of very long pulses with the advantages of very short pulses. The range resolution of a simple pulse-modulated radar depends on the pulse duration. Two reflective objects located within the spatial extent of the pulse are only displayed as one target. To improve the range resolution for a relatively long transmission pulse duration, the transmission pulse is modulated internally. Now a frequency comparison can be made in the received echo, for example, which makes it possible to localize the reflecting object within the pulse.

Several modulation methods can be applied. There are pulse compression methods:

- ❖ Frequency modulated (or called Frequency Modulation on Pulse, FMOP)
  - ✓ with linear frequency modulation,
  - ✓ with non-linear frequency modulation,
  - ✓ with time-dependent coded frequency modulation (e.g. the Costas code)
- ❖ Phase modulated (or called Phase Modulation on Pulse, PMOP)
  - ✓ with time-dependent coded pulse-phase modulation.

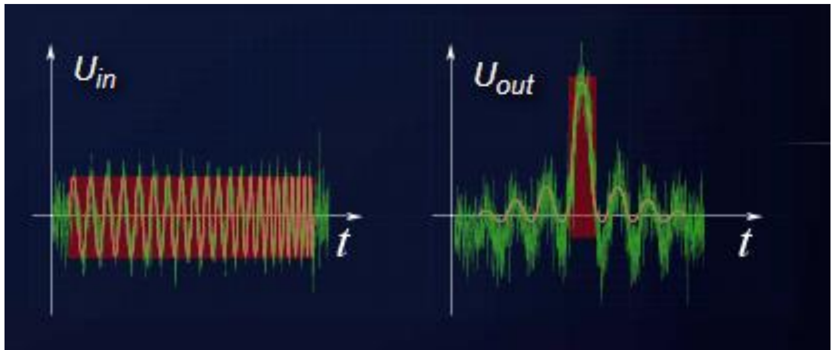



*Figure 1: Input and output signals of a pulse compression stage, the received signal in noise is hardly noticeable, so the pulse compression results in a clear echo signal.*

The Pulse Compression Ratio (PCR) is the ratio of the time length of the uncompressed transmitted pulse to the length of the compressed pulse.

The noise is always broadband and the noise pulses have a statistical distribution. The frequency-synchronous part of the noise (i.e. noise at the same frequency as the modulated received signal) is rather low compared to the echo signal. Therefore, the non-frequency synchronous part of the input noise is reduced by the filters. This way an output signal is still obtained even if the input signal has long since been lost in the noise and would thus be lost for simple demodulation. Compared to the non-modulated pulse, an additional gain is thus obtained, the pulse compression gain or pulse compression factor, which is approximately equal to the Pulse Compression Ratio (PCR).

For a linear (i.e. not divided into discrete single pulses) frequency modulation of the transmit pulse, the bandwidth B of the transmit pulse with the pulse duration $\tau$ is decisive. For further calculations, the time-bandwidth product is introduced, the derivation of which results from the ratio of the different range resolutions:

$$PCR = \frac{(c_0 \cdot \tau/2)}{(c_0/2B)} = B \cdot \tau$$

The range resolution of a pulse modulated radar is therefore a multiple (by a factor of the Pulse Compression Rate PCR) of the range resolution of an intra-pulse modulated radar:

$$R_{res} = c_0 \cdot (\tau/2) = PCR \cdot c_0/2\,B$$

### Pulse Compression Gain:

With the help of pulse compression, a relatively long transmission pulse with comparatively low peak power can achieve a better, longer range than the basic radar equation would suggest. This is because pulse compression can still detect echo signals that have already disappeared in the noise before pulse compression. The probability is very low that a noise pattern similar to the intra-pulse modulation will occur in such a way that this noise also forms an output signal during pulse compression.

In the radar equation, the advantage of intrapulse modulation and pulse compression must be seen as an increase in range. In the equation the pulse compression ratio PCR or N is often entered directly, i.e. the transmitted pulse length and the length of the compressed pulse. This then results in a pulse power multiplied by the transmission pulse duration, i.e. a transmission pulse energy. This is divided by the minimum possible received power PE min multiplied by the duration of the compressed pulse, together also an energy. The pulse compression ratio is sometimes also called Pulse Compression Factor K, because it is entered directly as a factor in the radar equation under the fourth root:

$$K = T \cdot B = N = \frac{T}{\tau_c}$$

Where; T = Length of transmitted pulse

B = Bandwidth of Transmitted pulse

$\tau_C$ = Length of compressed pulse

$$R_{\max} = \sqrt[4]{\frac{P_S \cdot T \cdot G^2 \cdot \lambda^2 \cdot \sigma}{P_{E_{\min}} \cdot \tau_c \cdot (4\pi)^3 \cdot L_{ges}}}$$

However, this requires a largely lossless pulse compression, which can never be achieved in practice. For this reason, it is better to use the quantity Pulse Compression Gain, which is to be determined by measurement and takes into account the conversion losses. Alternatively, the pulse compression loss can also be used separately (called loss for the mismatch of optimal filters Ln).

The disadvantage of this method, however, is that the blind range of the Chirp-radar is very much worse. As long as the transmitter is working, nothing can be received, because the duplexer blocks the receivers during this time. Only with the use of ferrite circulators is it possible to transmit and receive simultaneously. However, these ferrite circulators can only be used for relatively low transmitting powers.

## Advantages and Disadvantages of the Pulse Compression:

### Advantages

- **Lower pulse-power therefore suitable for Solid-State-amplifier**
- higher maximum range
- Good range resolution
- Better jamming immunity
- Difficulter reconnaissance

### Disadvantages

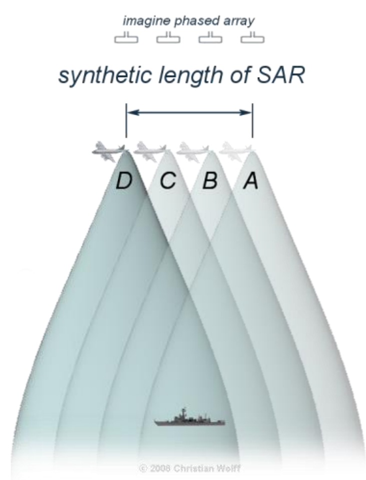
- **High wiring effort**
- Bad minimum range
- Time-sidelobes
- Filter inaccuracies due to Doppler frequencies

## Synthetic Aperture Radar:

A Synthetic Aperture Radar (SAR), or SAR, is a coherent mostly airborne or space-borne side-looking radar system which utilizes the flight path of the platform to simulate an extremely large antenna or aperture electronically, and that generates high-resolution remote sensing imagery. Over time, individual transmit/receive cycles (PRT's) are completed with the data from each cycle being stored electronically. The signal processing uses magnitude and phase of the received signals over successive pulses from elements of a synthetic aperture. After a given number of cycles, the stored data is recombined (taking into account the Doppler effects inherent in the different transmitter to target geometry in each succeeding cycle) to create a high-resolution image of the terrain being over flown.

### How does SAR works?

The SAR works similar of a phased array, but contrary of a large number of the parallel antenna elements of a phased array, SAR uses one antenna in time-multiplex. The different geometric positions of the antenna elements are result of the moving platform now.




The SAR-processor stores all the radar returned signals, as amplitudes and phases, for the time period T from position A to D. Now it is possible to reconstruct the signal which would have been obtained by an antenna of length $v \cdot T$, where v is the platform speed. As the line of sight direction changes along the radar platform trajectory, a synthetic aperture is produced by signal processing that has the effect of lengthening the antenna. Making T large makes the „synthetic aperture" large and hence a higher resolution can be achieved.

As a target (like a ship) first enters the radar beam, the backscattered echoes from each transmitted pulse begin to be recorded. As the platform continues to move forward, all echoes from the target for each pulse are recorded during the entire time that the target is within the beam. The point at which the target leaves the view of the radar beam some time later, determines the length of the simulated or synthesized antenna. The synthesized expanding beamwidth, combined with the increased time a target is within the beam as ground range increases, balance each other, such that the resolution remains constant across the entire swath.

The achievable azimuth resolution of a SAR is approximately equal to one-half the length of the actual (real) antenna and does not depend on platform altitude (distance).

The requirements are:

- Stable, full-coherent transmitter
- An efficient and powerful SAR-processor, and
- Exactly knowledge of the flight path and the velocity of the platform.

Using such a technique, radar designers are able to achieve resolutions which would require real aperture antennas so large as to be impractical with arrays ranging in size up to 10 m.

A Synthetic Aperture Radar was used on board of a Space Shuttle during the Shuttle Radar Topography Mission (SRTM).

SAR radar is partnered by what is termed Inverse SAR (abbreviated to ISAR) technology which in the broadest terms, utilizes the movement of the target rather than the emitter to create the synthetic aperture. ISAR radars have a significant role aboard maritime patrol aircraft to provide them with radar image of sufficient quality to allow it to be used for target recognition purposes.

## SAR Image Interpretation:

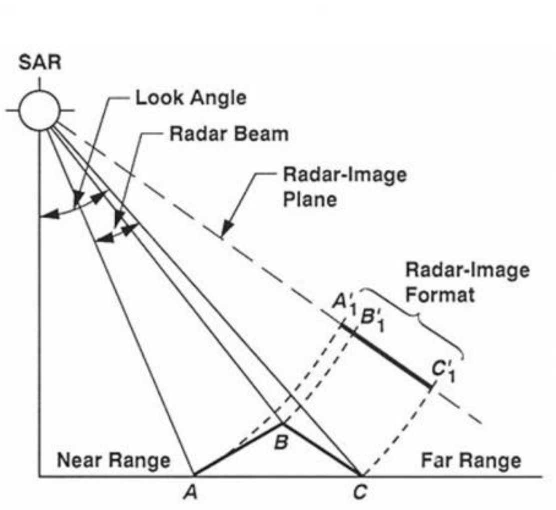
While the images created by SAR can be rendered into a recognizable terrain map, there are important differences between optical imagery and SAR imagery. SAR imagery is considered a non-literal imagery type because it does not look like an optical image which is generally intuitive to humans. These aspects must be understood for accurate image interpretation to be performed.

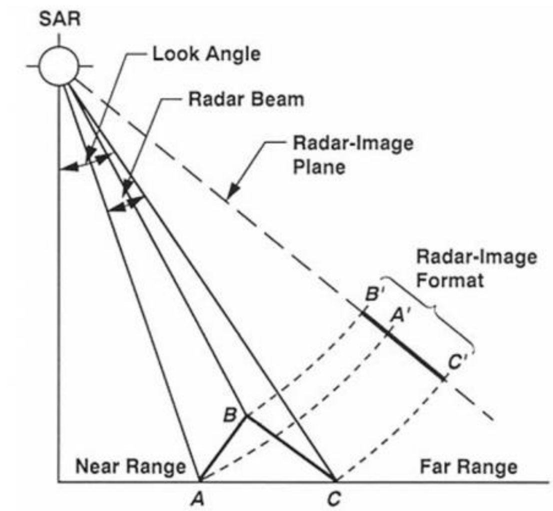
### *Shadowing*

Shadowing is caused for the same reasons that shadows are formed in optical imagery: an object blocks the path of direct radiation—visible light in the case of optical imaging and the radar beam in the case of SAR. However, unlike optical imagery in which objects in shadows can be seen due to atmospheric scattering, there is no information in a SAR shadow because there is no return signal.

### *Foreshortening*

Because SAR is a side-looking, ranging instrument, the backscattered returns will be arranged in the image based on how far the target is from the antenna *along the slant plane* (radar-image plane). This causes some interesting geometrical distortions in the imagery, such as foreshortening. As seen in Figure 4, the slope A-B is compressed in the slant plane because the radar signal reaches point B shortly after reaching point A in time. This causes a tall object with a slope, such as a mountain, to appear steeper, with a thin bright "edge" appearance. Note that the sensor's look angle affects foreshortening; a larger look angle will decrease the effect.



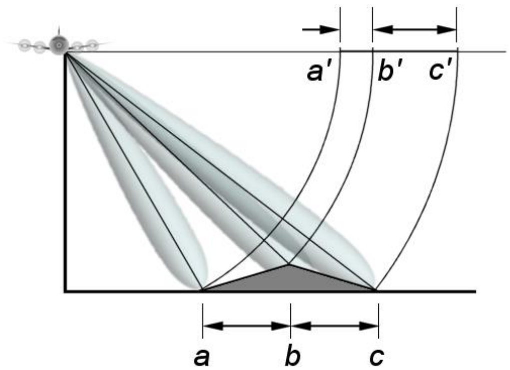

*Layover*

Layover is an extreme example of foreshortening where the object is so tall that the radar signal reaches point B before it reaches point A. This causes the returns from point B to be placed on the image closer to the sensor (near range) and obscure point A, as if the top has been overlaid on the foot of the mountain.

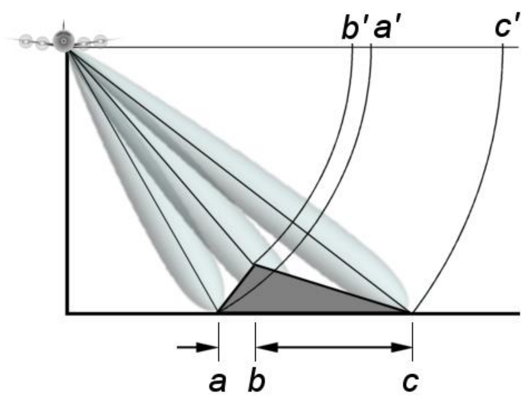
## Slant-range distortion:

The slant-range distortion occurs because the radar is measuring the distance to features in slant-range rather than the true horizontal distance along the ground. This results in a varying image scale, moving from near to far range.

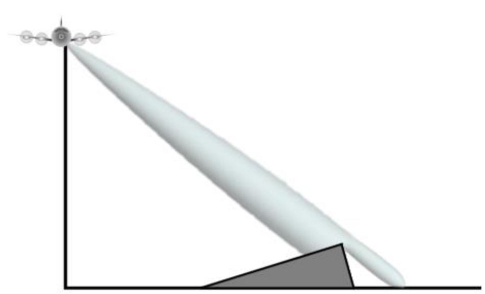
Foreshortening occurs when the radar beam reaches the base of a tall feature tilted towards the radar (e.g. a mountain) before it reaches the top. Because the radar measures distance in slant-range, the slope (from point a to point b) will appear compressed and the length of the slope will be represented incorrectly (a' to b') at the image plane.




Layover occurs when the radar beam reaches the top of a tall feature (b) before it reaches the base (a). The return signal from the top of the feature will be received before the signal from the bottom. As a result, the top of the feature is displaced towards the radar from its true position on the ground, and „lays over" the base of the feature (b' to a').




The shadowing effect increases with greater incident angle θ, just as our shadows lengthen as the sun sets.



## Probability of False alarm and Detection:

The results of statistical noise theory will be applied to obtain:

***The signal-to-noise ratio at the output of the IF amplifier necessary to achieve a specified probability of detection without exceeding a specified probability of false alarm*.**

The output signal-to-noise ratio thus obtained is substituted into the final modified radar equation, we have obtained earlier.

**The details of system that is considered**:

- **IF** amplifier with bandwidth $\mathbf{B_{IF}}$ followed by a second detector and a video amplifier with bandwidth **BV** as shown in the figure below.
- The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope.
- To extract the modulation envelope, the video bandwidth must be wide enough to pass the low-frequency components generated by the second detector, but not so wide as to pass the high-frequency components at or near the intermediate frequency.
- The video bandwidth **BV** must be greater than **BIF/2** in order to pass all the video modulation.

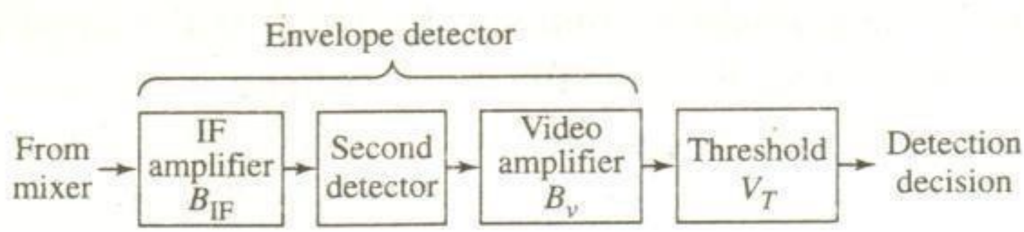



**Figure:** Envelope detector.

**Step 1: To determine the Probability of false alarm when noise alone is assumed to be present asinput to the receiver:**

The noise entering the IF filter (the terms filter and amplifier are used interchangeably) is

$$p(v) = \frac{1}{\sqrt{2\pi\psi_0}} \exp \frac{-v^2}{2\psi_0}$$

assumed to beGaussian, with probability-density function given by

Where

**p(v) dv** is the probability of finding the noise voltage ***v*** between the values of **v**and**v + dv**

***ψ0*** is the variance, or mean-square value of the noise voltage, and the mean value of **v** is taken to be zero.

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x-x_0)^2}{2\sigma^2}$$

*(Compare this with the Standard Probability density function of Gaussian noise With* $\sigma^2$ *replaced by* $\psi_0$*and **(x—x0)** replaced by **v** with mean value of zero)*

If Gaussian noise were passed through a narrowband IF filter whose Bandwidth is small compared with its mid band frequency-the probability density of the envelope of the noise voltage output is shown by **Rice** to be of the form of **Rayleigh** probability-density function

$$p(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right)$$

where ***R is the amplitude of the envelope of the filter output***. The probability that the envelope of thenoise voltage will lie between the values of ***V1***and***V2***is

$$\text{Probability } (V_1 < R < V_2) = \int_{V_1}^{V_2} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR$$

The probability that the noise voltage envelope will exceed the voltage threshold **VT** is

$$\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR$$

$$= \exp\left(-\frac{V_T^2}{2\psi_0}\right) = P_{fa}$$

Whenever the voltage envelope exceeds the threshold **VT**, a target is considered to have been detected. Since the probability of a false alarm is the probability that noise will cross the threshold, the above equation gives the probability of a false alarm, denoted by ***Pfa***.

The probability of false alarm as given above by itself does not indicate that Radar is troubled by the false indications of Target. The time between the false alarms **TFA** is a better measure of the effect of Noise on the Radar performance. (Explained with reference to the figure below)

The average time interval between crossings of the threshold by noise alone is defined as the

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k$$

F*alse- alarm time $T_{FA}$*

Where **TK** is the time between crossings of the threshold ***VT*** by the noise envelope, when the slope of the crossing is positive.

The false-alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it ***could have been*** above the threshold, i.e.

$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{av}}{\langle T_k \rangle_{av}} = \frac{1}{T_{fa} B}$$

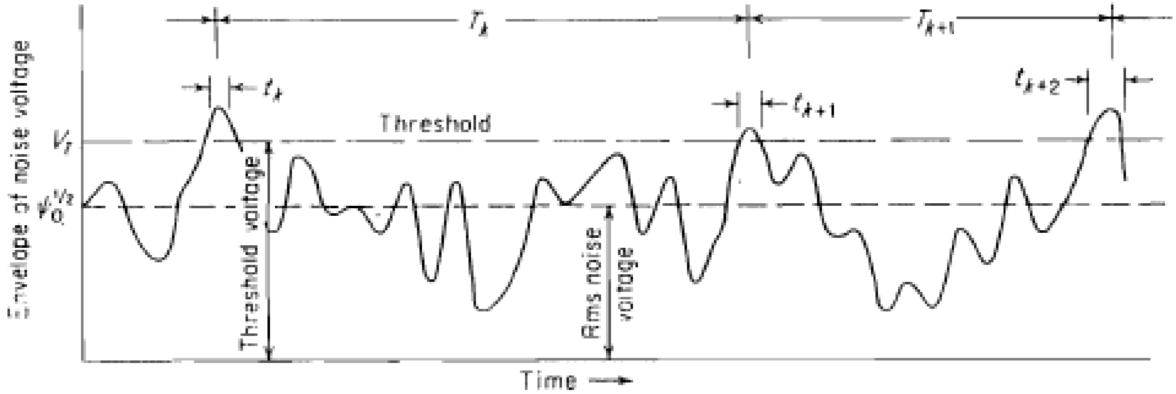



**Fig: Envelope of receiver output illustrating false alarms due to noise.**

Where $t_K$ and $T_K$ are shown in the Figure above. ***The average duration of a noise pulse is approximately the reciprocal of the bandwidth B,*** which in the case of the envelope detector is $B_{IF}$.

$$T_{fa} = \frac{1}{B_{IF}} \exp \frac{V_T^2}{2\psi_0}$$

..........(

A plot of the above equation is shown in the figure below with **($V^2/2\psi$)** as the abscissa. As can be seen, average time between false alarms $T_{fa}$ is directly proportional to the Threshold to noise ratio and inversely proportional to the Bandwidth.

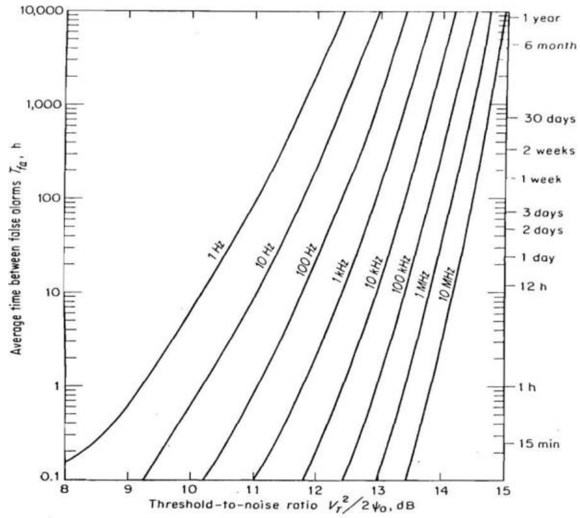



**Fig: Average time between false alarms as a function of the threshold level $V_T$ and the receiver Bandwidth B. $\psi_0$ is the mean square noise voltage**

### *Step 2 : To determine Probability of detection when a sine wave signal is present along with noise:*

Thus far, a receiver with only a noise input was discussed. Next, consider a sine-wave signal of amplitude **A** to be present along with noise at the input to the IF filters. The frequency of the signal is the same as the IF mid band frequency $f_{IF}$. The output of the envelope detector has a probability-density function given by

$$p_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right)$$

where $I_0(Z)$ is the modified Bessel function of zero order and argument **Z.**

When the signal is absent, $A = 0$ and the above equation for **PDF** for signal plus noise reduces to the probability-density function for noise alone. This Equation is sometimes called the **Rice** probability- density function.

The probability that the signal will be detected (which is the ***probability of detection)*** is the same as the probability that the envelope **R** will exceed the predetermined threshold ***$V_T$***. The probability of detection **$P_d$** is therefore:

$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) dR$$

(After the expression of **PDF** for **Ps(R)[Eq. 10]** is substituted into the first part of the above equation we get the probability of detection as in **[eqn.11]).** But this equation cannot be evaluated by simple means, and numerical & empirical techniques or a series approximation must be used.

The expression for **$P_d$** given by equation **(11)** after series expansion is a function of the signal amplitude A, threshold voltage $V_T$ ,and mean noise power $\Psi_0$ . In Radar systems analysis, it is more convenient to use Signal to Noise power ratio (S/N) rather than signal to noise voltage ratio $A/\Psi_0^{1/2}$.These are relatedby:

$$\frac{A}{\psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}(\text{rms signal voltage})}{\text{rms noise voltage}} = \left(2\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

The probability of detection $P_d$ can then be expressed in terms of S/N , and Threshold- Noise ratio $V_T^2/2\Psi_o$ .The probability of false alarm is also a function of $V_T^2/2\Psi_o$ as given by : **$P_{FA} = \mathrm{Exp}(-V_T^2/2\psi_o)$.**

The two expressions for $P_d$ and $P_{FA}$ can now be combined by eliminating the Threshold- Noise ratio **$V_T^2/2\Psi_o$** that is common in both expressions so as to get a single expression relating the probability of detection **$P_d$** ,Probability of false alarm **$P_{FA}$** and signal to Noise ratio **S/N**.The result is plotted in the figure below.

A much easier empirical formula developed by ***Albersheim*** for the relationship between **S/N,PFA** and **Pd** is also given below :

$$S/N = A + 0.12AB + 1.7\,B$$

**Where; A = ln [ 0.62/ PFA]**

**B = ln [ Pd /(1- Pd) ]**

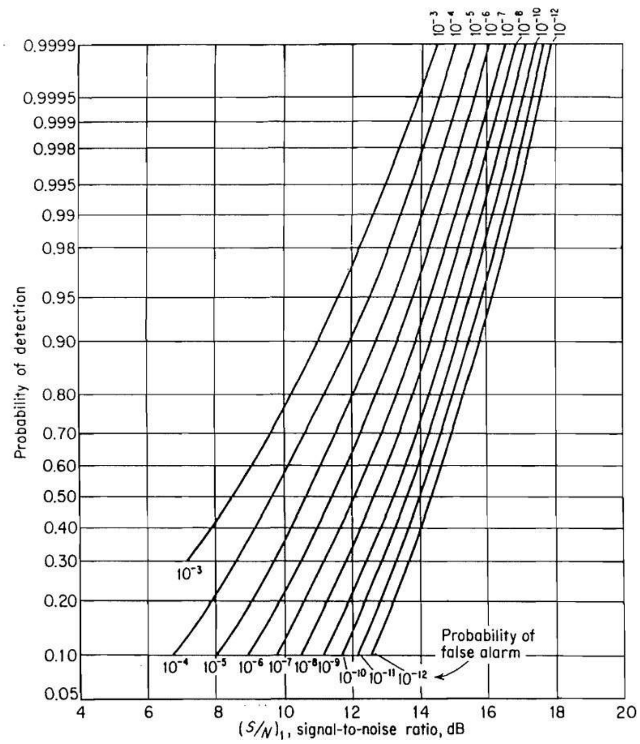



***Fig: Probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm***

# Unit – 05

## Ground Penetrating Radar:

Ground Penetrating Radar (GPR) is a geophysical survey method that uses pulses of electromagnetic radiation to image the subsurface. It provides a non-intrusive and non-destructive method of surveying the sub-surface. Consequently, it is a useful survey technique to investigate many materials. Examples include the ground, concrete, masonry, and asphalt.

GPR uses high-frequency (usually polarized) radio waves, usually in the range 10 MHz to 2.6 GHz. A GPR transmitter and antenna emits electromagnetic energy into the ground. When the energy encounters a buried object or a boundary between materials having different permittivities, it may be reflected or refracted or scattered back to the surface. A receiving antenna can then record the variations in the return signal.

Ground Penetrating Radar (GPR) uses high-frequency pulsed electromagnetic waves to map subsurface information. GPR uses transmitting and receiving antennae, which are dragged along the ground surface. The transmitting antenna radiates short pulses of high-frequency radio waves into the ground. The wave spreads out and travels downward. If it hits a buried object or a boundary with different electrical properties, the receiving antenna records variations in the reflected return signal. The principles involved are similar to reflection seismology, except that electromagnetic energy is used instead of acoustic energy, and the resulting image is relatively easy to interpret. Integration of GPR data with other surface geophysical methods reduces uncertainty in site characterization. GPR provides the highest lateral and vertical resolution of any surface geophysical method.

Best penetration is achieved in dry sandy soils or massive dry materials such as granite, limestone, and concrete. GPR provides the greatest resolution of currently available surface geophysical methods. It is the only reliable method for detecting buried plastic containers.

A GPR transmitter emits pulses of electromagnetic energy into the subsurface. Changes in the sub-surface are detected based on differences in permittivities. When a change in the sub-surface is encountered, some of the electromagnetic energy is reflected back to the surface. This is detected by a receiving antenna and variations in the return signal are recorded. The information is displayed on a radargram.

Although Ground Penetrating Radar can detect changes in the sub-surface, it can't determine their exact nature. Some features exhibit specific characteristics in the reflected wave pattern. For example, reflections from metallic surfaces have a high amplitude, while reflections from a void are reverse polarity. These help with the identification of the detected features. However, in some cases, it may be necessary to supplement a Ground Penetrating Radar survey with absolute data from boreholes, sample cores, trial pits, etc.

## What can Ground Penetrating Radar detect?

Ground Penetrating Radar (GPR) can effectively be used to locate and distinguish a wide variety of metallic and non-metallic materials. GPR works best when there is a big difference in the electromagnetic properties of the materials being surveyed. For this reason, metallic objects make ideal targets (e.g. reinforcement in concrete). Ground Penetrating Radar will detect most materials providing there is a sufficient difference in the electromagnetic property between the target and surrounding material. Some of the more common target materials include:

- Metal
- Plastics
- Changes in ground strata and geological features
- Concrete
- Air pockets or voids

Excavated areas, back-filled areas and any other ground disturbances can also be identified and mapped.

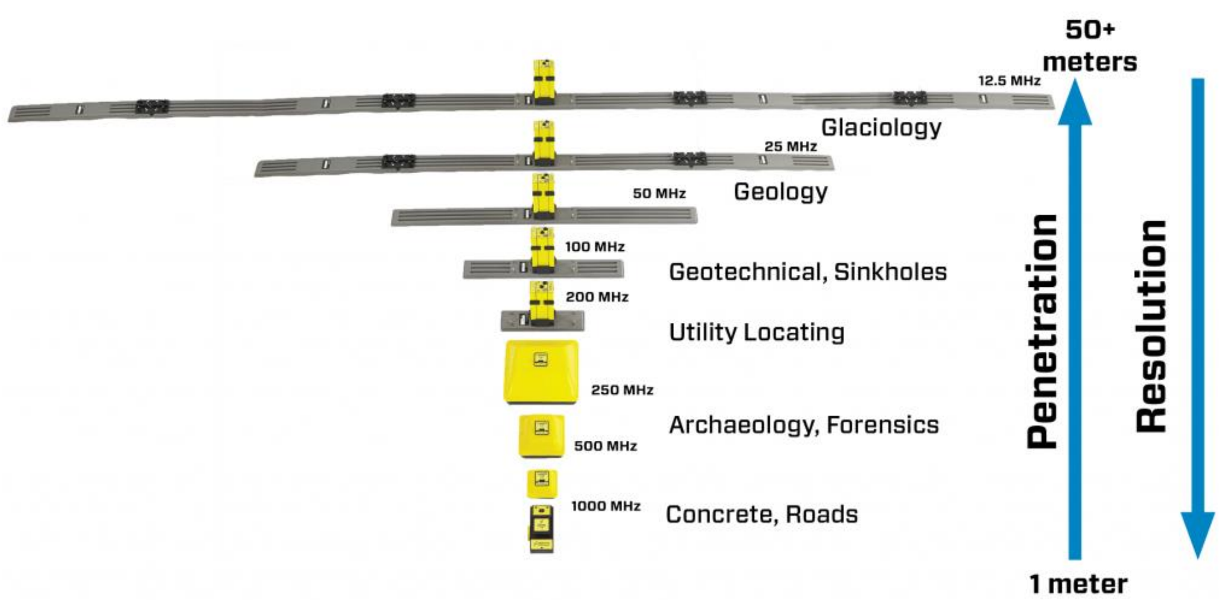
Ground Penetrating Radar will not work in certain ground conditions such as heavy clay soils, particularly if they are waterlogged. De-ionised water does not pose a problem to GPR. However, water with a high mineral content (e.g. seawater) attenuates the signal making it an unsuitable medium. Ground Penetrating Radar is also unable to penetrate through metallic objects, including very dense reinforcement.

## Ground Penetrating Radar Penetration Depth:

The electrical conductivity of the scanned medium, the transmitted centre frequency, and the radiated power all influence the penetration depth.

An increase in electrical conductivity attenuates the GPR electromagnetic wave. This results in reduced penetration depth.

Higher frequencies provide a higher resolution; however, the penetration depth is limited. Conversely, a lower frequency provides greater penetration depth, albeit at a lower resolution. The choice of antennae frequency is dependent on the investigation objectives, including the width of the survey path. For example, an antenna frequency of 400 MHz has a 0.3 m survey path width. Generally, it is advisable to use the highest frequency possible. For best results, it is often necessary to scan with more than one frequency.



## Advantages of Ground Penetrating Radar:

Ground Penetrating Radar is a highly cost-effective non-disruptive survey technique. It offers a rapid means of obtaining subsurface information. Its many advantages include:

- GPR is non-intrusive, non-destructive and benign, making it safe for use in public spaces.
- It detects metallic and non-metallic objects and voids.
- It can resolve construction layer interfaces.
- Estimation of depth, dimensions of larger objects and layer thickness.
- Site data collection is relatively quick making it suitable for scanning of large areas.
- Only single-sided access is needed making it ideal for surveying floors, walls, decks, slabs, tunnels and balconies.
- Different frequencies provide different resolutions and penetration depths.
- High-resolution continuous survey data which can be interpreted qualitatively, in real-time, or processed off-site.
- Faster, safer and lower cost than radiography (X-ray).

## Limitations and Concerns:

Depth of penetration (typically 1 to 15 meters) is less than direct current (DC) resistivity and electromagnetic (EM) methods, and is further reduced in moist and/or clayey soils and soils with high electrical conductivity. Penetration in clays and in materials having high moisture is sometimes less than 1 meter.

The GPR method is sensitive to noise—i.e., interference caused by various geologic and cultural factors. For example, boulders, animal burrows, tree roots, and other phenomena can cause unwanted reflections or scattering. Cultural sources of noise can include reflections from nearby vehicles, buildings, fences, power lines, and trees. Electromagnetic transmissions from cellular telephones, two-way radios, television, and radio and microwave transmitters may also cause noise on GPR records. Shielded antennae are used to limit these types of reflections.

The bulkiness of equipment can limit use in rough terrain.

Unprocessed images require processing, as they provide only approximate shapes and depths (continuous microwave methods are still in developmental stages).

## Radar Tomography:

**Tomography** is imaging by sections or sectioning that uses any kind of penetrating wave. The method is used in radiology, archaeology, biology, atmospheric science, geophysics, oceanography, plasma physics, materials science, astrophysics, quantum information, and other areas of science. The word *tomography* is derived from Ancient Greek τόμος *tomos*, "slice, section" and γράφω *graphō*, "to write" or, in this context as well, "to describe." A device used in tomography is called a **tomograph**, while the image produced is a **tomogram**.

Computer-aided tomography is normally a process by which a 2D cross-sectional image of an object is obtained by illuminating it from many different directions in a plane. For the case of radar imaging, microwave energy reflected by the object is processed to produce an image which maps the object's radar cross-section (RCS) density into the image plane. Each observation provides a 1D projection of the RCS density.

Radar, and in particular imaging radar, has many and varied applications to security. Radar is a day/night all-weather sensor, and imaging radars carried by aircraft or satellites are routinely able to achieve highresolution images of target scenes, and to detect and classify stationary and moving targets at operational ranges. Different frequency bands may be used, for example high frequencies (X-band) may be used to support high bandwidths to give high range resolution, while low frequencies (HF or VHF) are used for foliage penetration to detect targets hidden in forests, or for ground penetration to detect buried targets. The techniques of tomographic imaging were originally developed in the context of medical imaging, and have been used with a number of different kinds of radiation, both electromagnetic and acoustic. The purpose of this presentation is to explore the application of tomographic imaging techniques at RF frequencies to a number of different applications in security, ranging from air defence to the detection of concealed weapons. Of particular interest is the use of ultra narrow band (UNB) transmissions with geometric diversity in a multistatic configuration to image moving targets. In the limit such transmissions could be CW, which would be particularly attractive for operation in a spectrally congested environment. This arrangement effectively trades angular domain bandwidth for frequency domain bandwidth to achieve spatial resolution. Also of interest is the improvement in target classification performance afforded by multi-aspect imaging.

## Tomographic Imaging:

The techniques of tomography were developed originally for medical imaging, to provide 2D cross-sectional images of a 3D object from a set of narrow X-ray views of an object over the full 360◦ of direction. The results of the received signals measured from various angles are then integrated to form the image, by means of the Projection Slice Theorem. The Radon Transform is an equation derived from this theorem which is used by various techniques to generate tomographic images. Two examples of these techniques are Filtered Backprojection (FBP) and Time Domain Correlation (TDC). Further descriptions of these techniques may be found in [20]. In radar tomography the observation of an object from a single radar location can be mapped into Fourier space. Coherently integrating the mappings from multiple viewing angles enables a three dimensional projection in Fourier space. This enables a three dimensional image of an object to be constructed using conventional tomography techniques such as wavefront reconstruction theory and backprojection where the imaging parameters are determined by the occupancy in Fourier space. Complications can arise when target surfaces are hidden or masked at any stage in the detection process. This shows that intervisibility characteristics of the target scattering function are partly responsible for determining the imaging properties of moving target tomography. In other words, if a scatterer on an object is masked it cannot contribute to the imaging process and thus no resolution improvement is gained. However, if a higher number of viewing angles are employed then this can be minimised.

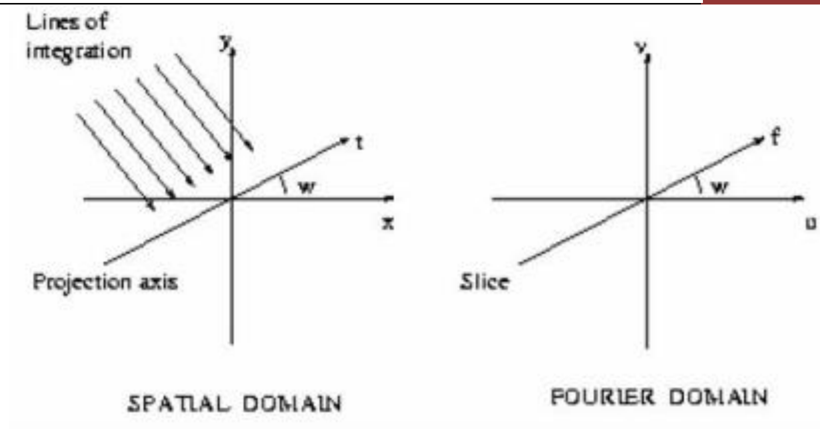



**Figure:** Tomographic reconstruction: the Projection Slice Theorem.

Further complications may arise if (a) the point scatterer assumption used is unrealistic (as in the case of large scatterers introducing translational motion effects), (b) the small angle imaging assumption does not apply and (c) targets with unknown motions (such as non-uniform rotational motions) create cross-product terms that cannot be resolved.

## The Projection Slice Theorem:

The Tomographic Reconstruction (TR) algorithm makes use of the Projection-Slice theorem of the Fourier transform to compute the image. The Projection-Slice theorem states that the 1D Fourier transform of the projection of a 2D function g(x, y), made at an angle w, is equal to a slice of the 2D Fourier transform of the function at an angle w, see Figure 3. Whereas some algorithms convert the outputs from many radars simultaneously into a reflectivity image using a 2D Fourier transform, TR generates an image by projecting the 1D Fourier transform of each radar projection individually back onto a 2D grid of image pixels. This operation gives rise to the term Backprojection. The image can be reconstructed from the projections using the Radon transform. The equation below shows this:

$$g(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} P(f)\cdot|f|\cdot e^{j2\pi f(x\cos w+y\sin w)}df\,dw$$

Where; w = projection angle

P(f) = the Fourier transform of the 1-D projection p(t).

The Filtered Back projection (FBP) method may be used to process by reconstructing the original image from its projections in two steps: Filtering and Back projection. Filtering the projection: The first step of FB Preconstruction is to perform the frequency integration (the inner integration) of the above equation. This entails filtering each of the projections using a filter with frequency response of magnitude |f|. The filtering operation may be implemented by ascertaining the filter impulse response required and then performing convolution or a FFT/IFFT combination to correlate p(t) against the impulse response. Back projection: The second step of FB Preconstruction is to perform the angle integration (the outer integration) of the above equation. This projects the 1D filtered projection p(t) onto the 2D image by following these steps: place a pixel-by-pixel rectangular grid over the XY plane, then place the 1D filtered projection p(t) in position at angle w for each pixel, then get the position of the sample needed from the projection angle and pixel position. Interpolate the filtered projection to obtain the sample. Add this back projection value multiplied by the angle spacing. Repeat the whole process for each successive projection.

## Microwave Imaging:

**Microwave imaging** is a science which has been evolved from older detecting/locating techniques (e.g., radar) in order to evaluate hidden or embedded objects in a structure (or media) using electromagnetic (EM) waves in microwave regime (i.e., ~300 MHz-300 GHz).[1] Engineering and application oriented microwave imaging for non-destructive testing is called **microwave testing**, see below.

Microwave imaging techniques can be classified as either quantitative or qualitative. Quantitative imaging techniques (are also known as inverse scattering methods) give the electrical (i.e., electrical and magnetic property distribution) and geometrical parameters (i.e., shape, size and location) of an imaged object by solving a nonlinear inverse problem.

Microwave testing is a useful NDT method for dielectric materials. Among them are plastics, glass-fiber reinforced plastics (GFRP), plastic foams, wood, wood-plastic composites (WPC), and most types of ceramics. Defects interior in the DUT and at its surface can be detected, e. g. in semi-finished products or pipes.

### *Special applications* of microwave testing are non-destructive

- Moisture measurements
- Wall thickness measurements
- Measurements of paint thickness on carbon composites (CFRP)
- Condition monitoring, e. g. presence of gaskets in assembled valves, rubber based piping in heat exchangers
- Measurement of material parameters, e.g. permittivity and residual stress

### *Microwave testing is used in many industrial sectors:*

- Aerospace, e. g. non-destructive paint thickness measurements on CFRP
- Automobile, e. g. NDT of organo sheet components and of GFRP leaf springs
- Civil engineering, e. g. radar applications
- Energy supply, e. g. test of rotor blades of wind power plants, riser pipe
- Security, e. g. body scanner on airports